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Optimal tariffs and retaliation with perfect foresight

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Optimal tariffs and retaliation with perfect foresight

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CHAPTER 1. REVIEW OF THE LITERATURE

The theory of optimal tariffs essentially took form with the writings of Edgeworth in 1894 (1925), Bickerdike (1906), and Kaldor (1940). Although it is well-known that--under domestic free market conditions--free trade represents a Pareto equilibrium for the world as a whole, the theory of optimal tariffs attempts to show that free trade is not necessarily the optimal welfare position for a country that possesses some degree of monopoly power with respect to commodity prices. That is, if a country is economically large enough to affect its terms of trade, then the theory of optimal tariffs concludes that it is possible for that country to increase its welfare by imposing some tariff--assuming that the other country does not retaliate. The graphical analysis of this theory is covered in most international trade textbooks. For example, in Figure 1.1, point E represents free trade equilibrium. U_1' represents the highest indifference curve that country one can reach, given that country two's offer curve stays where it is. This indifference

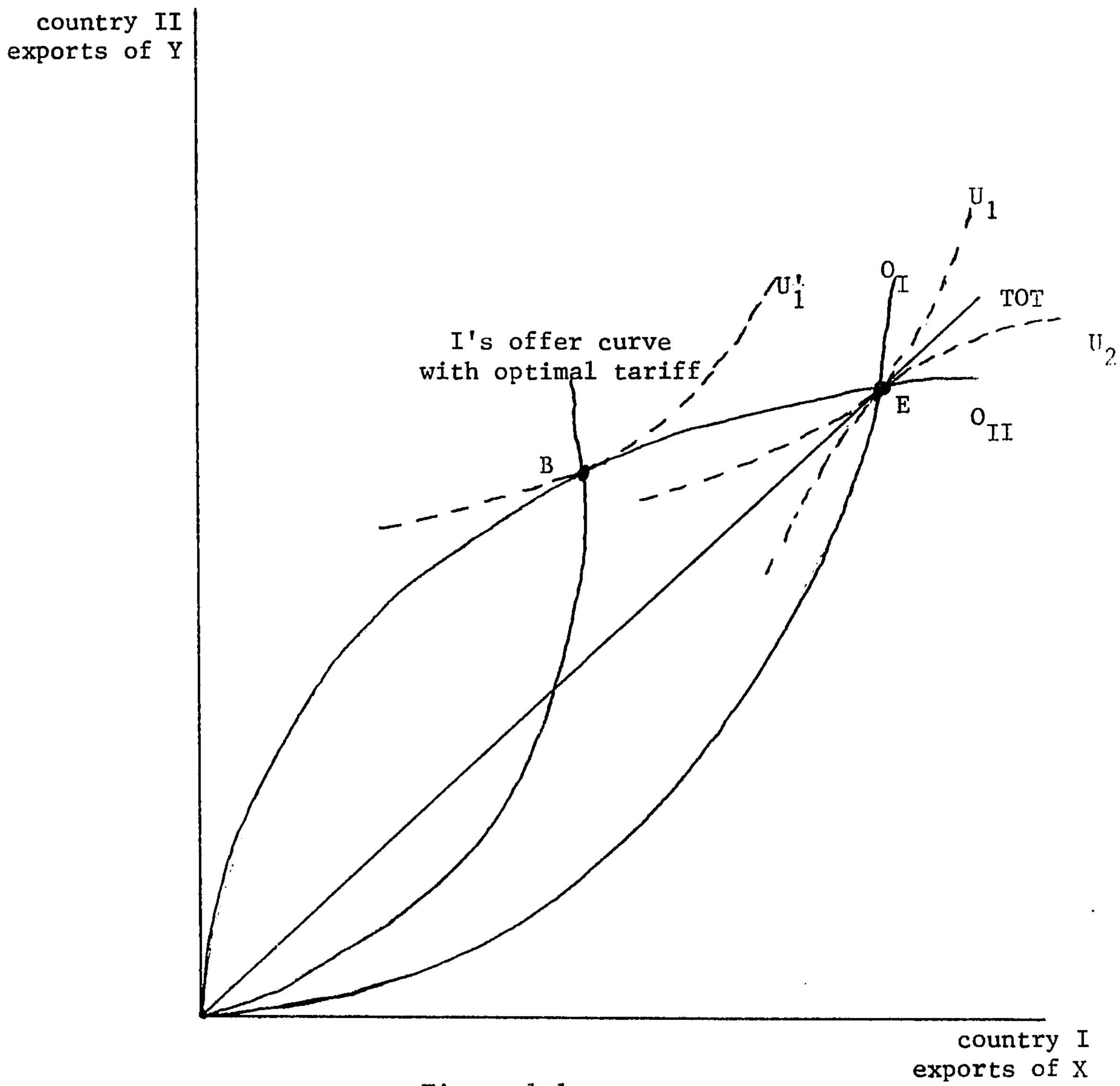

Figure 1.1
Optimal tariff with no retaliation

curve is tangent to the given offer curve at point B, so country one would place the appropriate tariff to shift its offer curve through point B. For a good presentation of the mathematical derivation of the optimal tariff with no retaliation, see Takayama (1972), which parallels the original work by Graff (1949). The conclusion of the theory is that if only an import tariff is used, and there is no retaliation, the formula for the optimal _ad valorem_ tariff (T) is

$$T = \frac{1}{e - 1} \tag{1.1}$$

Where (e) is the elasticity of the foreign country's offer curve.

Of course, if a country does impose its optimal tariff, some country now has a lower welfare level than it did before, and it seems plausible that the country adversely affected would consider some form of retaliation. In an effort to increase the explanatory power of the theory, several attempts have been made to revise this critical assumption.

The initial models, such as those of Johnson (1953), used a Cournot (equivalently a Stackelberg--two follower) type of argument in which a country imposing an optimal tariff naively assumes that the other country will not alter its terms of trade in response to the imposition of the tariff. Most of the models also postulate that the tariff proceeds will be redistributed to consumers--either directly or indirectly.

Using these assumptions, Johnson derives reaction curves for each country which represent the optimal tariff given the other country's current tariff policy. They are derived as follows :

> The tangency points of one country's indifference curves with the other's offer curves trace out loci of optimal-tariff equilibrium points (welfare-reaction curves) for the respective countries.

A simple example of such a reaction curve is shown in figure 1.2. Country two is presumed to accept country one's offer curve as given, hence, indifference curves are shown tangent to several possible country one offer curves (at the B_i points). By connecting these points, the reaction curve R_{II} is derived. Note that the curve does not have to be as "nice" as it is drawn here. For example, it could change curvature several times. Using these curves, Johnson demonstrates that two situations can arise. Either some equilibrium point will be reached at an intersection of the reaction curves (after some indeterminate number of retaliations by each country), or a tariff cycle will result in which some country applies its optimal tariff, the second country

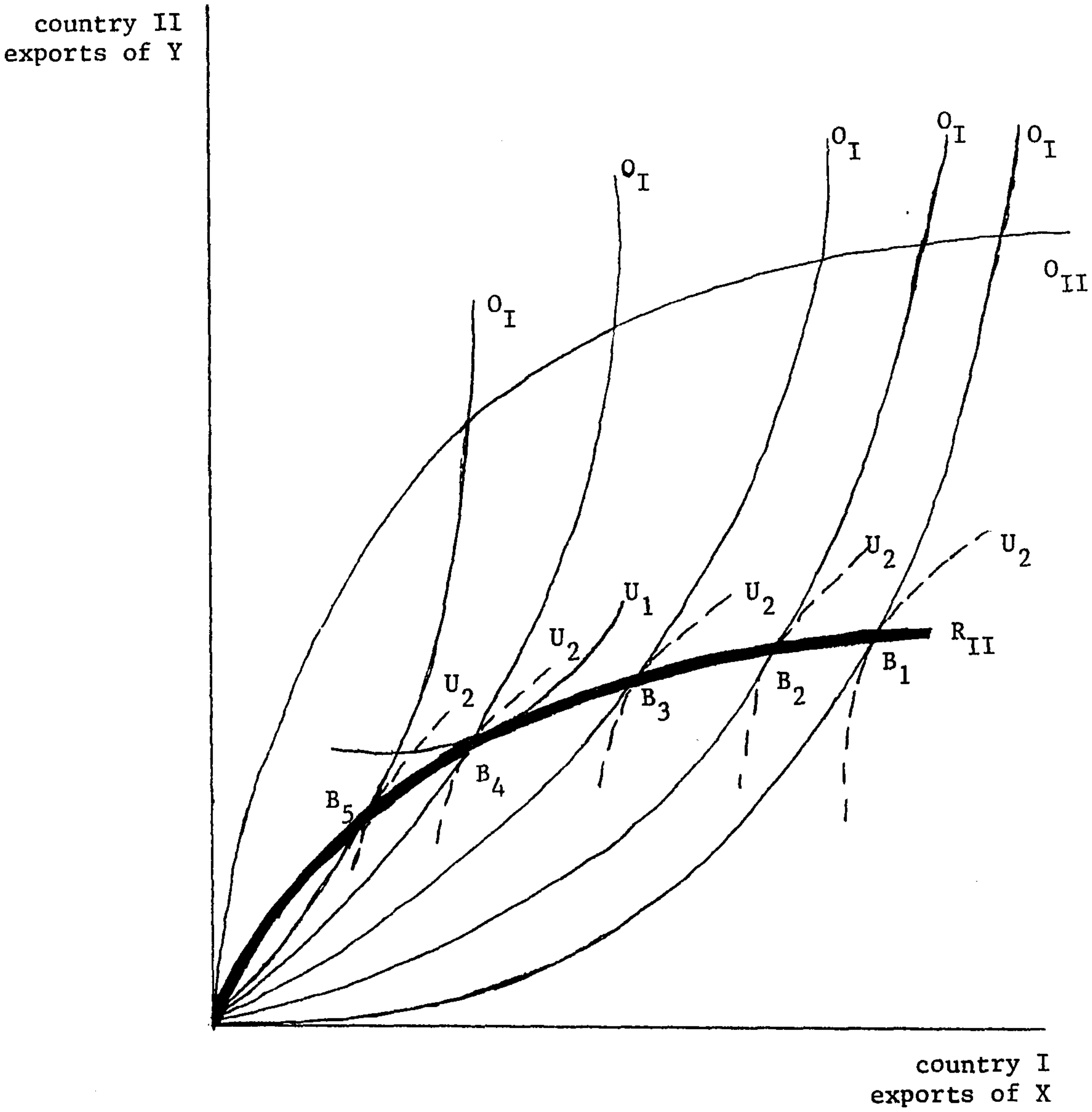

Figure 1.2
Optimal tariff--derivation of reaction curve

retaliates, and the first country's optimal welfare position now necessitates a reduction in the tariff rate, causing country two to decrease its tariff rate. Johnson assumes that both countries consider themselves to be Cournot followers, and stay on their reaction curves, leading (for example) to an equilibrium at point B in figure 1.3. Or, this tariff cycle would continue indefinitely. The final welfare conclusion (in both cases) is that either country, but not both, could end up with a higher level of welfare than it had initially, or both countries might have lower levels of welfare.

Using a similar type of analysis, Horwell (1966) analyzed the difference between _ad valorem_ and _specific_ tariffs. His conclusions are that with the Cournot model of retaliation, the reaction curve generated by an _ad valorem_ tariff lies further out than one generated by a _specific_ tariff. See figure 1.4. Hence, in most cases, an _ad valorem_ tariff will yield a higher level of welfare for the initiating country than would a _specific_ tariff.

Gorman (1958) analyzes the elasticity of demand for imports to determine "the effects of tariffs on the volume and terms of trade," and "the conditions under which a given country will gain from a tariff war." He

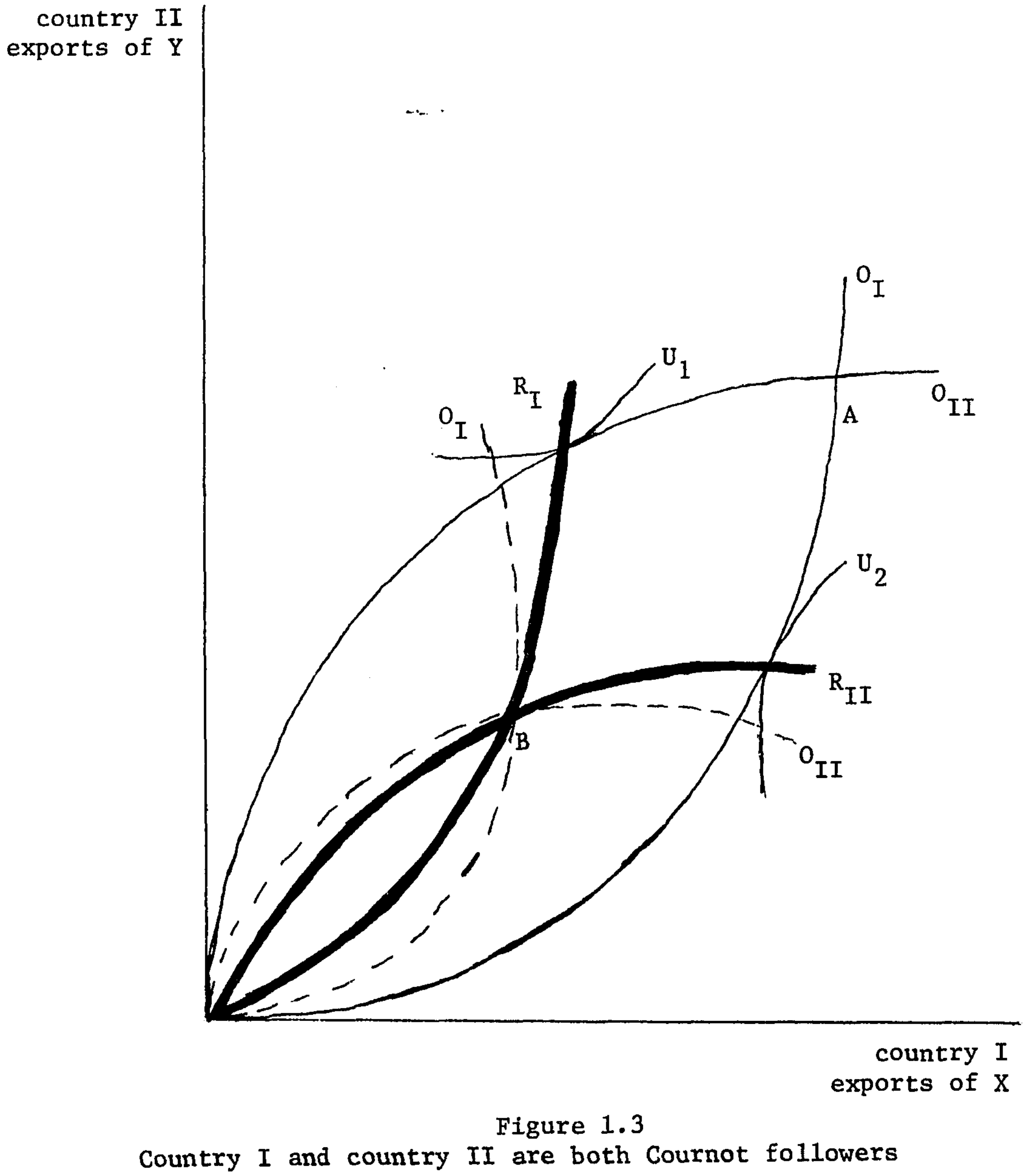

Figure 1.3
Country I and country II are both Cournot followers

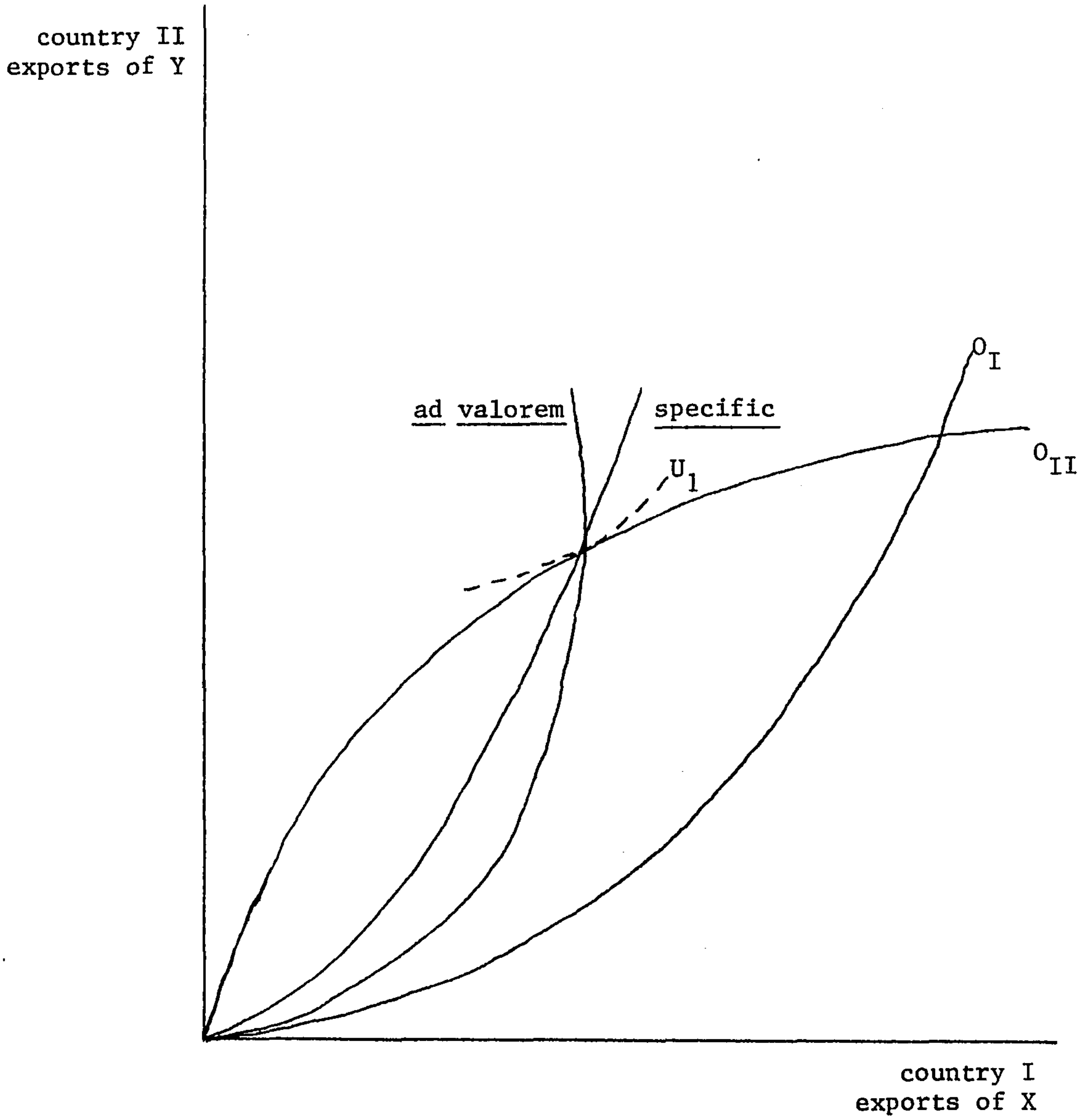

Figure 1.4
Reaction curves for ad valorem and specific tariffs

then goes on to approximate how much the volume of trade would decrease in a tariff war, under different elasticity of import conditions. Similarly, he presents a range of elasticity conditions under which a country could conceivably gain from engaging in a tariff war.

Rodriquez (1974) demonstrates that with a Cournot model, in a two country, two good model, optimal import quotas are not equivalent to optimal tariffs. In fact, "optimal quota retaliation will lead to the elimination of international trade between the countries involved." Tower (1975) demonstrates the same conclusion when both countries are using export quotas. For example, in figure 1.5, country two acts first and imposes an optimal export quota to force the equilibrium point to point A.

Country two's new offer curve is denoted by Oa'a. Country one now perceives that they can achieve the same level of imports for a smaller level of exports, so they impose a quota, yielding offer curve Oba'. Responding to this new offer curve, country two will impose a more restrictive tariff. This process will continue until trade goes to zero. Tower arrives at the same conclusion (trade goes to zero) if one country imposes an optimal tariff and the other an optimal quota.

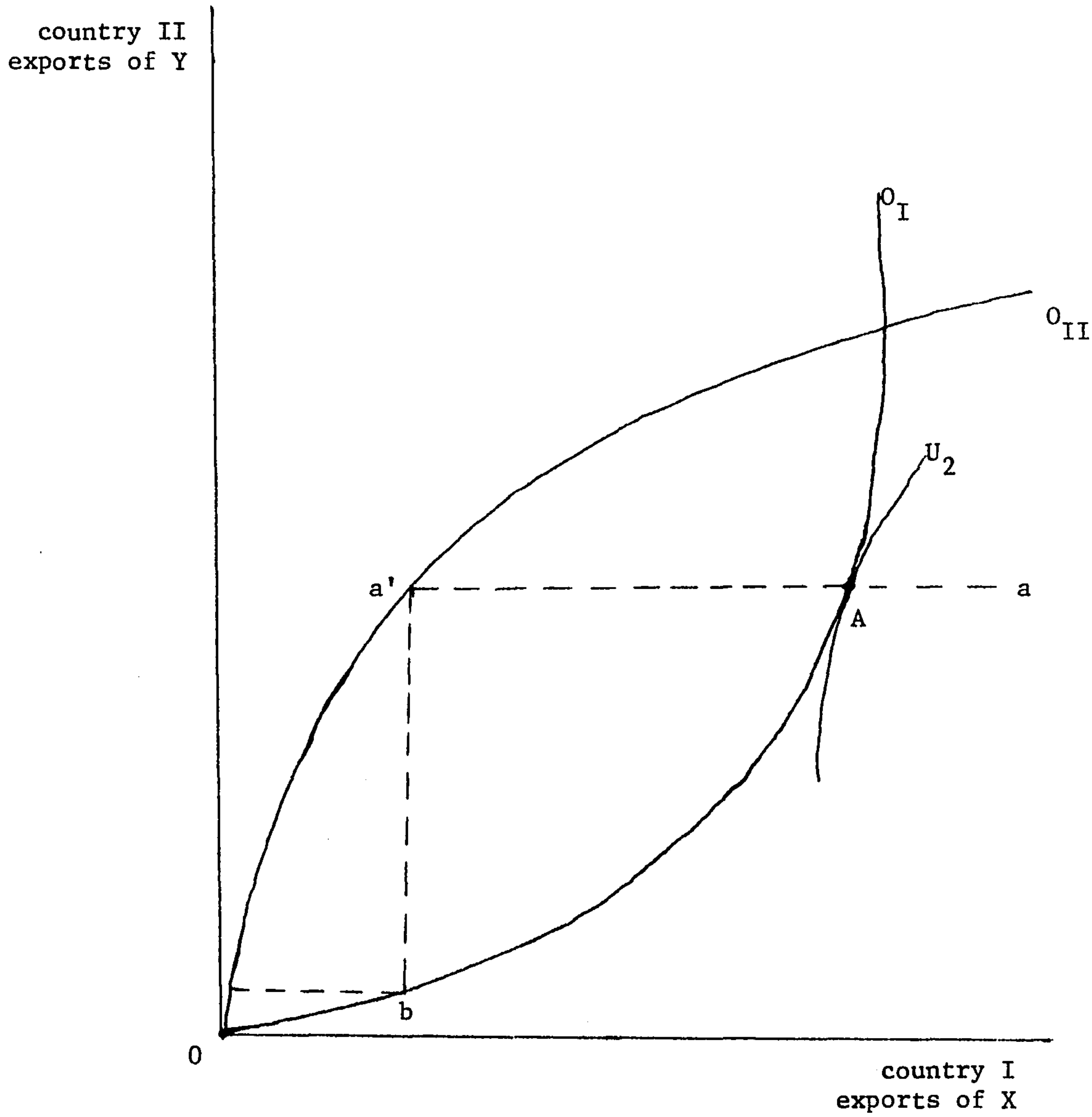

Figure 1.5
Both countries use optimal export quotas

Some attention has been paid to the assumption that the tariff revenue is redistributed to consumers. Bertrand (1973) considers the case where the revenue is used by the government, and he maximizes a government welfare function dependent on an export good and an import good. By maximizing a Langrangean expression of welfare subject to balance of trade equilibrium in both countries, he shows that the optimal tariff rate is

$$T1 = e_I e_{II}/(e_I - 1)(e_{II} - 1) \tag{1.2}$$

where the e_I and e_{II} are the elasticities of the foreign and domestic private sector offer curves respectively.

Tower (1977) considered the case where the tariff revenue is redistributed to consumers, but instead of maximizing a welfare function, he maximizes the tariff revenue, and demonstrates that if the home trade indifference curves are strictly concave, and the foreign offer curve has a continuous first derivative, then the maximum revenue tariff will be larger than the optimal tariff.

The early work of Graff (1949) presenting a multi-commodity welfare maximization case was extended by Vandendorpe (1972), and more recently by Ghosh (1979). Using the notation of Ghosh, for n commodities, with tariffs a_i $(i = 1,n)$, for some set of tariffs unalterable, the optimal values for the controllable tariffs are found by maximizing a utility function subject to a concave transformation surface, balance of goods market, and market clearing conditions. Letting E_i represent excess demand for the ith commodity, the optimal tariffs are

$$a_h = -(\Sigma\Sigma D_{ih}(dE_j/da_i)a_j)/|D| \qquad (1.3)$$

where $|D|$ is the determinant (dE_j/da_i), for $i,j = m+1,\ldots,n$ and D_{ih} is the cofactor of the i-hth element of that determinant. This equation reduces to $a_h = \Sigma b_j a_j$ where $\Sigma b_j = 1$. Both Vandendorpe and Ghosh were also concerned with second-best analysis. That is, they attempt to derive the optimal tariff and consumption and production taxes if some institutional constraints on the use or level of tariffs or taxes are present. The early work of Vandendorpe (1968) was extended (within a two country-two commodity case) to three tax instruments, with two arbitrarily fixed, by Dornbusch (1971). For E_2 representing the elasticity of the foreign offer curve, P the domestic price, and MSC the marginal social cost of an additional unit of exports, then the optimal tariff is shown to be

$$T = \frac{1}{E_2 - 1} - \frac{P-MSC}{P} \frac{E_2}{E_2 - 1} \tag{1.4}$$

The first part is the traditional optimal tariff formula and the second part is derived from a weighted average of domestic taxes. Similarly, Markusen (1975) was interested in a second-best optimal intervention. He specifically covered a model of two countries related by a "bilateral production externality." For example, he covers a case where consumption and production taxes are arbitrarily constrained to zero. In this case, the optimal tariff is the sum of four terms. The first term is simply the standard optimal tariff. The second term is "due to the existence of domestic pollution," and is always negative. It will lower the optimal import tariff relative to the foreign elasticity of supply. The third term is based on the fact that the "consumer's marginal rate of substitution may depend upon the total flow of pollution," and the sign is ambiguous. The fourth term is "due to the existence of the foreign pollution externality," and is always positive. Essentially, "the gains of exploiting monopoly power in trade can only be bought at the expense of an increase in domestic production," and hence, pollution. Similarly, Gehrels (1971) considered the relationship between optimal tariffs and optimal taxes on investment, and the second-best conclusions when one of the interventions is constrained. Using three factors of production (land, labor, and capital), Gehrels maximizes welfare as a function of consumption subject to the production constraint, and balance of trade. By maximizing with respect to exports, and the amount of foreign investment, the optimal tariff is shown to be

$$T = \frac{1}{N}\left(1 + \frac{Fp}{X_2 r}\,\frac{Ep}{Er}\right) \tag{1.5}$$

Note that r and p are the international terms of trade and lending respectively. N is the elasticity of demand for home-country exports of good 2; Fp/X2r is the "ratio of investment income to value of good 2 traded; and Ep/Er is the elasticity of the interest rate with respect to the terms of trade." In general, the sign of the bracketed term is ambiguous, but in the particular case that Gehrels examines; where the home country exports capital intensive good 2, and is a net exporter of capital; an increase in the tariff rate increases r, which decreases home production of good 2 and the rest of the world uses its capital more intensively, causing an improvement in the terms of lending (p). Therefore, the optimal tariff is higher for such a country when foreign investment is considered. Similarly, the optimal tariff is higher for a debtor country which also imports the capital-intensive good. Batra (1973) extends Gehrels' analysis in the presence of a wage differential. Boadway, et al. (1973) analyze a model where there is an exported good, an imported good, and a public good. Using this model, they derive the optimal tariffs (for both imports and exports), and show that the tariff formulae are the same as the traditional optimal tariff, but since domestic taxes or public goods may affect the demand elasticities, the actual rates may be different from the traditional model. They also consider the second-best condition of non-taxable domestic goods. For the import tariff T1 and the export tariff T2, the following equations are derived from a modified Langrangean:

$$1+T_1 = p(1+1/n_1) + (k_1/P_1)(dr_1/dX_1) \tag{1.6}$$

$$1+T_2 = p(1+1/n_2) + (k_2/P_2)(dr_2/dX_2) \tag{1.7}$$

where p is the Langrangean multiplier from the balance of trade constraint, P_1 and P_2 are the foreign prices (measured in domestic currency) of the importable and exportable good respectively, n_1 and n_2 are respectively the elasticities of excess demand for the exportable and importable good, k_1 and k_2 are the langrangean multipliers from the constraints that the two goods X_1 and X_2 are not domestically taxable, and r_1 and r_2 are the producer's price of the two goods. In this model, they show that the optimal import tariff is lower, and the optimal export tariff is higher than in the traditional case, since tariffs are now the only source of revenue available to provide the public goods.

Fishelson and Flatters (1975) added to the optimal tariff literature by introducing uncertainty into the model. They consider a model with a linear import supply curve, and a linear domestic demand curve. They first analyze a demand curve with a stochastic intercept term, and they conclude that an optimal tariff is superior to an optimal quota in that the welfare loss is lower with the tariff. Although they do not formally evaluate the case, they reach the same conclusion if the slope of the demand curve is stochastic. When they consider a linear model with a deterministic demand curve, and a stochastic intercept on the supply curve; or a stochastic slope on the supply curve; they conclude that if the supply curve of imports is elastic, then a tariff is preferred to a quota. However, if the supply curve is inelastic, then a quota may (but not necessarily will) be preferred to a tariff. Young (1979) extends the analysis and provides a more formal proof that an optimal quota can be preferred to an optimal tariff under uncertainty. The basic conclusion is that it is possible for an optimal quota to be preferred to an optimal tariff under uncertainty, if the degree of uncertainty of the supply elasticity (linear curves) is sufficiently small.

Tower (1975) was one of the first writers to relax the assumption of no retaliation. Using the reaction curves derived by Johnson (1953), Tower considers the optimal tariff from the perspective of country 1, when country 1 assumes that country 2 will automatically place an optimal tariff, based on the "current" trade situation. That is, in Stackelberg terminology, country 1 is a leader, and country 2 is a follower. Therefore, a reaction curve can be generated based on the action taken by country 1 that country 2 will always follow. Essentially, country 2's reaction curve is the _loci_ of the "points of tangency between one of 2's trade indifference curves and the given tariff-distorted offer curve of 1." For example, if both countries use tariffs and country one is a Stackelberg leader and country two is a follower, point A in Figure 1.2 represents the optimal tariff for country one to apply. Tower's basic conclusion is that "either the leader or the follower may be better off than in free-trade equilibrium." He also notes that a Stackelberg leader will always prefer an optimal tariff to an optimal quota.

To date, the most general relaxation of the non-retaliation assumption is the case considered by Bhagwati and Srinivasan (1976). They also deal with a case in which country 1 is a leader, and country 2 is a follower. Essentially, they are interested in the optimal tariff that country one should impose, given that there is some probability that country two may impose an import quota in the next period. In other words, country one "knows" that country two will retaliate, but country two does not know that country one chooses a tariff based on two's reactions. Note that country one only knows the probability function that describes two's reaction. That is, they do not know exactly how two will respond. The import quota is a fixed level denoted E'. There is a known probability function P(E) that this quota will be imposed in the second period. This function is assumed to be convex in E; which means that as the level of exports gets larger, the probability that the quota will be imposed increases at an increasing rate. Domestic production possibilities are denoted by $F(X_1,X_2)$. Welfare is measured by a social welfare function $U(C_1,C_2)$. Welfare in the next period is denoted Uq if the quota is imposed, and Un if the quota is not imposed. Mathematically, the problem is solved by maximizing a two-period, two-good social utility function for country one with respect to the level of exports, and the two commodities; subject to the domestic transformation constraint. That is, country one maximizes

$$U(X_1-E,X_2+rE) + p(UqP(E)+Un(1-P(E))) \qquad (1.8)$$

subject to $F(X_1,X_2) = 0$. Carrying out the maximization yields

$$U_1/U_2 = (r+r'E)-(p(Un-Uq)/U_2)\ P'(E). \qquad (1.9)$$

In these equations, U_1 and U_2 are the first order derivatives with respect to goods X_1 and X_2, r is the terms of trade function, P(E) is the probability of a quota E' being imposed next period, and p is the one period discount factor. The first term of the optimal tariff in equation (1.9) reduces to the traditional no-retaliation optimal tariff. The second term is the discounted expected loss in welfare converted to numeraire terms. That is, if an additional unit of export takes place in period one, the probability of a quota being imposed increases by P'(E). Thus, the second term is the expected loss in welfare "at the margin."

The basic conclusion is that when such a probabilistic quota situation exists, country one needs to impose a tariff in order to maximize its utility level. As would be expected, if a quota is imposed, country one is better off if they impose a tariff. Also, free trade (no quota and no tariff) is preferred to the situation where country two imposes the quota and one does not retaliate. Finally, there is no way to determine if country one is better off with the quota and the retaliatory tariff or free trade. Note that although a degree of uncertainty has been introduced, these conclusions are not markedly different from the analysis in the previous literature.

It is fairly straight-forward to extend the analysis to a model in which production levels (X_1 and X_2) are fixed in the first period. That is, production cannot be modified at all in the second period, which economically means that a quota in period two could be very harmful if production was too high in period one. In this case, the optimal policy is to impose a consumption tax (tariff) in the first period to take advantage of the monopoly power in trade, and a production tariff/subsidy in the first period to account for the fixed production possibility. In the second period, it is only necessary to impose a tariff if the quota is imposed.

Bhagwati and Srinivasan also extend the analysis to a steady-state, infinite time horizon problem. In this case, once the quota is imposed, it is never removed. Secondly, the probability of it being imposed is only dependent on the level of exports in the previous period, and this probability is independent of time. Because of these assumptions, the results are very similar to those generated for the two period case. However, it is interesting to examine the levels of welfare that are generated from three cases. The highest level of welfare is achieved with free trade (no quota and no tariff). The lowest level of welfare (for country one) occurs when country two imposes the quota, and country one does not impose a retaliatory tariff. Falling between these two cases is the situation where country two does impose the quota, and one retaliates and imposes the optimal tariff.

This work provided an interesting analysis of uncertainty, where country one does not completely know how country two will respond. However, country two still incorrectly believes that its quota will not affect country one's tariff decision. Also, no attempt is made to explain where the probability function comes from, and why it remains unchanged in the infinite time horizon case. That is, after imposing a tariff in a period, each country will be able to examine the other's reaction, and reformulate their own beliefs about how the other country actually responds.

Kemp and Ohta (1978) add some generalizations to Bhagwati and Srinivasan's analysis by considering an infinite horizon, continuous time model. They note that the Bhagwati-Srinivasan case assumes

> that the probability that the quota will be imposed at any point of time, given that it has not been imposed already, depends only on the rate of flow of exports at that time. This is the polar case of a perishable, non-storable commodity.

Hence, besides offering a more formal analysis of that case, Kemp and Ohta consider the other polar case of a durable good, "with the probability depending on cumulative exports."

Kuga (1973) extended the earlier naive model of Johnson (1953), into a multi-country, multi-commodity framework. Each country has "import tariff policies that are country-wise and commodity-wise discriminative," while maximizing a social welfare function. Tariff revenue received is assumed to be redistributed to the private sector in a lump-sum fashion. Kuga then establishes the appropriate conditions (including a finite number of tariff options) necessary to apply the "Nash theory of noncooperative game to show the existence of mixed policy equilibrium." He then generates a two-commodity, three-country example to demonstrate the outcome of various tariff strategies on welfare levels in the three countries.

Otani (1980) takes a slightly different approach to the optimal tariff problem. Government agents in his model have incomplete information on domestic preferences, and on supply (both domestic and world--e.g. foreign tariff rates). Using the information available, government agents

> choose a system of tariffs to maximize the estimated preferences of domestic consumers with a constraint on an estimated availability of commodities.

This multi-commodity, multi-country model also assumes that any action by a government agent is based on the assumption that current tariff vectors will remain constant. That is, the agent assumes that no other countries will retaliate. Otani uses this model to show the general existence of a world equilibrium.

Tower, Sheer, and Baas, (1978) use numerical techniques to demonstrate "what happens to domestic and foreign welfare" under four different retaliation assumptions--ranging from the no retaliation assumption to a Stackelberg leader/follower assumption. They use an equivalent variation measure to evaluate the effects of different tariff rates under different elasticities of demand for imports.

CHAPTER 2. NOTES ON PERFECT FORESIGHT

All of the literature on optimal tariffs has relied on restrictive assumptions about the nature of retaliation (or lack of it). That is, at least one country maintains a naive assumption that the other will not retaliate--regardless of any "actual" change in tariffs. The essential question that remains to be answered is to determine if there are any solutions in which all countries involved know how the others will respond. In other words, both countries are aware that their actions will elicit a reaction from the other country.

A more precise way to state the problem is to consider that when each country attempts to maximize welfare (with respect to its tariff rate), there may be some change in the other country's tariff rate. Using this method, each country can then consider their optimal tariff rate as a function of what they believe is the other country's reaction. This reaction could be called the conjectural variation. The existing literature is a special case of this approach in that the anticipated reaction for at least one of the countries is zero. For the Cournot case, both countries assume that there will be no reaction. In the leader/follower case, the follower assumes there will be no reaction, and the leader then considers the follower's calculated response to the leader's tariff to derive its optimal tariff. In the Bhagwatti-Srinivasan case, the follower's reaction (as perceived by the leader) is a probabalistic function of the leader's exports.

Given that each country's optimal tariff is a function of the other's perceived reaction, there are essentially two ways to solve for the optimal tariff. First, each country could possess perfect foresight. That is, the perceived response function is equivalent to the actual response function. Second, relaxing this assumption slightly, each country may possess only imperfect foresight where the perceived response function is not necessarily known with certainty. That is, a country's belief could be wrong. Then, there must be a way to correct this belief.

The basic model assumes that there are two countries, trading two commodities. Each country produces, consumes and trades each commodity under free trade initially. Welfare is a function of consumption of each good, and government agents attempt to maximize some Social Welfare function. Production takes place on a Product Transformation curve that is negatively sloped and strictly concave. Each country's domestic economy is organized in a competitive manner, whereas the balance of goods and services is always in equilibrium. Finally, tariff revenue is redistributed to consumers in the form of additional income.

To keep the problem to a manageable level, consider a two-good, two-country world, where, in a partial equilibrium setting, welfare can be approximated by the consumer surplus or producer surplus generated from the excess supply and demand curves for one of the commodities. By maximizing the appropriate surplus, the optimal tariff for each country can be found as a function of the perceived reaction of the other country.

The focus of the problem now is to define the manner in which each country perceives the other's reaction. First, consider the possibility that each country possesses perfect foresight. Before beginning, it is important to understand what is meant by the term perfect foresight. Heuristically, a country can be said to possess perfect foresight if, for any given tariff that it could place, the country knows what tariff the other country will impose. On the surface, the condition could be expressed mathematically by noting that the level of the other country's tariff must equal the expected value of the tariff, which is the familiar Nash equilibrium condition. However, such a statement does not explain where their actual tariff comes from. That is, for perfect foresight to exist--for a country to know what tariff the other will impose--that country must know the process that the other uses to determine what tariff it will impose. In essence, to have perfect foresight, a country must know the reaction function of the other country; where the reaction function is an expression of the tariff that the second country will impose for any given tariff of the first country. One possible approach to the problem is to hypothesize that each country actually follows some reaction function dependent on the other country's tariff. Each country also knows the other's reaction, and knows that the other country also possesses perfect foresight. One possible approach to the problem is shown by Bresnahan (1981), in which he attempts to find the solution to a similar problem posed for the case of a duopoly.

The Bresnahan paper treats a case of two producers, where each is cognizant that a change in their output will affect the output of the other firm which will create a secondary affect on the market price, which affects the first firm's profits. Hence, when firm i attempts to maximize profits, it considers the impact of the change in output of firm j. That is, differentiating the profit function for the first firm entails the inclusion of a reaction term. Using an inverse demand function, and assuming perfect substitutes, profit for firm one can be written as

$$PI_1 = P_1(q)q_1 - c_1(q_1) \tag{2.1}$$

where $q = q_1+q_2$. The first-order condition for profit maximization is

$$\begin{aligned} 0 = q_1[\partial P_1(q_1,q_2)/\partial q_1 + \partial P_1(q_1,q_2)/\partial q_2 \cdot \\ r_{12}(q_1)] + P_1(q_1,q_2) - \partial c_1(q_1)/\partial q_1 \end{aligned} \tag{2.2}$$

Note that r_{12} is the conjectural variation. That is, r_{12} is what country one believes will be country two's response to a change in q_1. Bresnahan then notes that (2.2) can be solved for q_1 as a function of q_2 and r_{12}. Call this function p_1. A similar process will yield q_2 as a function of q_1 and r_{21}, denoted p_2, where r_{21} is two's perception of how country one will react to changes in q_2. He then defines a perfect foresight condition, which he calls a consistent conjectures equilibrium (CCE), as a point in output space (q_1*,q_2*) such that $q_1* = p_1(q_2*)$ and $q_2* = p_2(q_1*)$, which is the Nash equilibrium. Further, to be consistent, the conjectural variations must equal what he calls the actual variation. Or, in his notation,

$$r_{12}(q_1) = dp_2(q_1)/dq_1 \qquad (2.3)$$

$$r_{21}(q_2) = dp_1(q_2)/dq_2 \qquad (2.4)$$

which must hold for all (q_1,q_2) within an epsilon neighborhood of the Nash equilibrium (q_1*,q_2*). The left-hand side in equations (2.3) and (2.4) are the conjectural variations, and the right-hand sides are what Bresnahan (1981) considers to be the actual variations. To solve the system, he contends that if the reaction functions are constrained to be polynomials, then they must be linear, hence, the second derivatives of r_{12} and r_{21} must be zero. As a result, when equation (2.2) is solved for $q_1 = p_1(q_2,r_{12})$, it can be differentiated with respect to q_2, and dr_{12}/dq_2 will drop out, making the solution much easier. Bresnahan then proposes to substitute the resulting formulas for dp_2/dq_1 and dp_1/dq_2 into the p_1 and p_2 functions to determine the equilibrium (q_1*,q_2*). Before analyzing some of the objections to his analysis, it should be pointed out that the method is easily transferred to the international trade situation of two large countries.

Let T_1 and T_2 be the tariffs imposed by countries one and two respectively. If each country is maximizing some welfare function (or a relevant surplus), then the welfare is a function with respect to T_1 and T_2, or

$$U_1 = U_1(T_1, T_2') \tag{2.5}$$

$$U_2 = U_2(T_1', T_2) \tag{2.6}$$

where T_2' is one's belief about what equilibrium T_2 will be, and T_1' is defined symmetrically. Differentiating (2.5) and (2.6) to determine the first-order conditions yields

$$0 = \partial U_1/\partial T_1 + (\partial U_1/\partial T_2)r_{12} \tag{2.7}$$

$$0 = \partial U_2/\partial T_2 + (\partial U_2/\partial T_1)r_{21} \tag{2.8}$$

Now, equations (2.7) and (2.8) can be solved respectively to yield

$$T_1 = G_1(T_2', r_{12}) \tag{2.9}$$

$$T_2 = G_2(T_1', r_{21}) \tag{2.10}$$

Which means that T_1 can be expressed as a function of T_2 and country one's conjectural variation about $\partial T_2/\partial T_1$, and a similar statement can be made about T_2. Without imposing rationality at the moment, consider an example that Bresnahan presents. That is, it seems reasonable to ask if the Cournot assumption could represent perfect foresight equilibrium.

In the Cournot case, both countries are behaving as followers in the sense that they accept the other country's tariff as given, and do not expect it to change as they change their own tariff. Therefore, both r_{12} and r_{21} are identically equal to zero. Imposing this constraint would mean that equations (2.9) and (2.10) would reduce to the following

$$T_1 = G_1(T_2) \tag{2.11}$$

$$T_2 = G_2(T_1) \tag{2.12}$$

As long as neither country alters its belief, equations (2.11) and (2.12) will represent the true reactions of country one and two respectively to changes in the other's tariff.

To illustrate the process, consider a two-country, two-good world in which country one exports good X. P represents the price of good X in country one. Country one imposes a specific export tariff (T_1) on good X, and country two imposes a specific import tariff (T_2) on good X. Assume that the excess demand curve for X can be described by the general function

$$D: \quad Qd = Qd(P+T_1+T_2) \tag{2.13}$$

and the general excess supply curve in free trade can be described by

$$S: \quad Qs = Qs(P) \tag{2.14}$$

Equilibrium quantity is found by simultaneously solving equations (2.13) and (2.14) for Q, yielding

$$Qe = Qe(T_1,T_2) \tag{2.15}$$

As shown in Figure 2.1, the net producers' surplus (exclusive of tariff revenue) accruing to country one (from the exports only) can be represented by area A. Similarly, the net consumers' surplus accruing to country two from the imports can be represented by area C. B_1 and B_2 represent the tariff revenue received by countries one and two respectively. The objective of country one is to maximize the sum of areas A and B_1 with respect to T_1. Country two attempts to maximize the sum of C and B_2 with respect to T_2. Let U_1 equal $A+B_1$, then U_1 can be written as

$$U_1 = T_1Qs(P) + \int_{a_1}^{Pe} Qs(P)dP \tag{2.16}$$

Differentiating with respect to T_1, yields

$$dU_1/dT_1 = Qe + T_1S'dPe/dT_1 + QedPe/dT_1 \tag{2.17}$$

Where dPe/dT_1 comes from the equilibrium condition

$$Qs(P) = Qd(P+T_1+T_2) \tag{2.18}$$

Totally differentiating each side yields

$$dPe = D'/(S'-D')\ (dT_1+dT_2) \tag{2.19}$$

Letting $r_{12}(T_1)$ be the the conjectural variation that is held by country one, substituting dPe/dT_1 into equation (2.17), and solving for T_1 yields

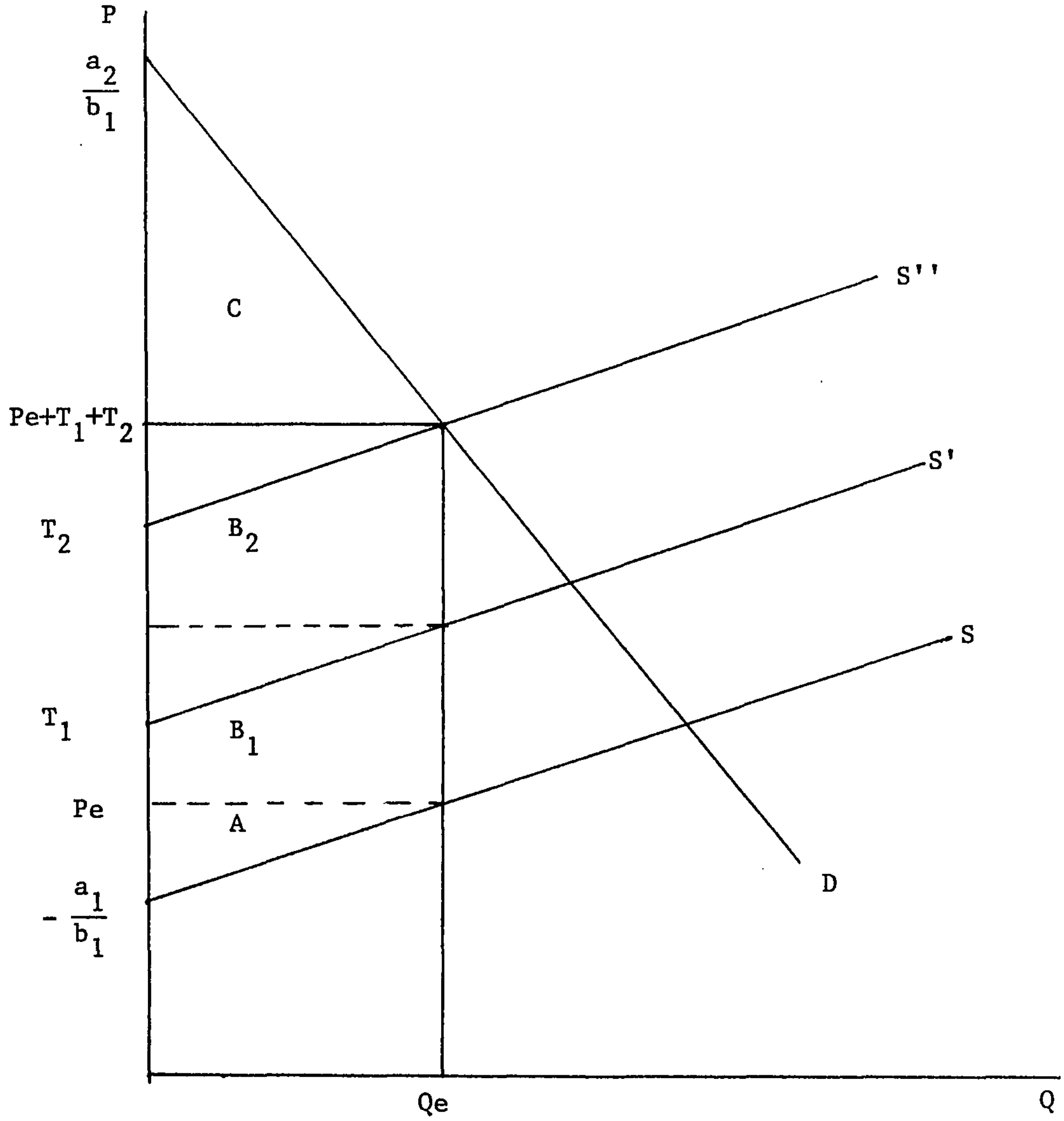

Figure 2.1
Producer and consumer surplus from trade

The intercepts come from assuming linear supply and demand curves:

S: $Q = a_1 + b_1P$

D: $Q = a_2 - b_1(P+T_1T_2)$

$$T_1 = \frac{-Qe(S'+D'r_{12})}{S'D'(1+r_{12})} \qquad (2.20)$$

Similarly, from the perspective of country two, for U_2 equal to C plus B_2,

$$U_2 = T_2Qd(Pe+T_1+T_2) + \int_{Pe+T_1+T_2}^{a_2} Qd(P)dP \qquad (2.21)$$

This equation can similarly be solved for T_2

$$T_2 = \frac{Qe(D'+S'r_{21})}{S'D'(1+r_{21})} \qquad (2.22)$$

Consider a simplified case where both the excess supply and demand curves are linear. Then,

$$S: \quad Q = a_1 + b_1P \qquad (2.23)$$

$$D: \quad Q = a_2 + b_2(P+T_1+T_2) \qquad (2.24)$$

Note that to obtain a downward sloping demand curve and a positive slope on the supply curve, the following relations must hold: $a_1 < 0$, $b_1 > 0$, $a_2 > 0$, $b_2 < 0$, $a_2b_1 - a_1b_2 > 0$. Substituting (2.23) into (2.20), and solving for T_1 yields

$$T_1 = \frac{(a_2b_1-a_1b_2+b_2b_1T_2')(b_1+b_2r_{12})}{b_2b_1(b_2-b_1(2+r_{12}))} \qquad (2.25)$$

Similarly,

$$T_2 = \frac{(a_2b_1-a_1b_2+b_2b_1T_1')(b_2+b_1r_{21})}{b_2b_1(b_1-b_2(2+r_{21}))} \qquad (2.26)$$

where T_2' represents one's belief about the level of two's tariff, and T_1' represents two's belief about the level of one's tariff.

As noted above, the Cournot case implies that $r_{12} = r_{21} = 0$. That is, neither country expects changes in their tariff to affect the other country's tariff. Substituting zero into equations (2.25) and (2.26) for r_{12} and r_{21} yields

$$T_1 = \frac{(a_2b_1 - a_1b_2 + b_2b_1T_2')}{b_2(b_2 - 2b_1)} \tag{2.27}$$

and

$$T_2 = \frac{(a_2b_1 - b_1b_2 + b_2b_1T_1')}{b_1(b_1 - 2b_2)} \tag{2.28}$$

Note that equation (2.27) defines the best T_1 for country one to impose for any value of its belief about T_2'. Equation (2.28) symmetrically defines the optimal T_2 for any belief T_1'. Now, as is well-known, there is a Nash equilibrium where $T_1 = T_1'$ and $T_2 = T_2'$, which is the familiar equilibrium after the tariff wars run their course. At this Cournot equilibrium point, equation (2.27) will represent one's reaction curve with respect to a tariff imposed by country two. The important point to note is that if equation (2.27) is differentiated with respect to T_2, the result is non-zero. This result is in conflict with the original belief by country two that r_{21} is identically equal to zero. That is, it contradicts the belief that country one would not respond to changes in two's tariff. Hence, perfect foresight does not prevail at the Cournot equilibrium. Bresnahan arrives at a similar conclusion for the duopoly case in that the Cournot assumption does not yield a consistent conjectures equilibrium.

In a similar manner, it can be shown that a Stackelberg leader/follower case can be represented as a perfect foresight case. If country one believes that country two is a follower, then they believe that equation (2.25) holds, and that country two uses the maximization process described above to find the optimal tariff, hence, equation (2.26) should hold. Country one further believes that country two thinks it is a follower, therefore, from one's perspective, r_{21} should be zero, which means that equation (2.26) reduces to

$$T_2 = \frac{(a_2b_1-a_1b_1+b_2b_1T_1)}{b_1(b_1-2b_2)} \tag{2.29}$$

Differentiation of (2.29) yields the reaction of country two to changes in one's tariff, or

$$r_{12} = dT_2/dT_1 = b_2/(b_1-2b_2) \tag{2.30}$$

Equation (2.30) is the reaction of country two that country one expects will occur. Substituting this result and the value for T_2 from (2.26) into equation (2.25) will yield the optimal tariff for country one to impose, which will be a function solely of the parameters of the model.

If country two actually considers itself to be a follower, and assumes that country one is a leader, then two really will behave in the fashion indicated in (2.30), and T_1 will be imposed as described. As a result, it is true that at the equilibrium point, the change in the believed reaction function T_1 with respect to T_2 will be zero--as indicated in the reduced version of equation (2.25).

However, what happens if country two does not want to be a follower? That is, given that country one is a leader and two is a follower, the above analysis defines the optimal tariff for each country to impose. A more general approach would allow each country to maximize its utility regardless of its current status as a leader or follower, and that status (if any) should follow as a result of the maximization process.

Judging by the above examples, it would seem to be a straightforward exercise to extend the analysis to both sides possessing perfect foresight, while still maximizing their respective surplus. If the analysis were extended along the lines proposed by Bresnahan (1981), equations (2.25) and (2.26) would be said to define the true reaction functions of each country, and the true responses could then be evaluated from these functions. The most general approach would be to claim that r_{12} and r_{21} are first derivatives of unknown reaction functions, and that in perfect foresight, we could assign equations (2.25) and (2.26) to be equal to the unknown functions. The result would be a system of two non-linear partial differential equations. Presumably this system could be solved for the two unknown functions, and once they are found, the equilibrium values for T_1 and T_2 could be found. However, such an approach would err in the same way that Bresnahan's approach errs.

Consider a system such as the one outlined above, or one such as Bresnahan's. Assume for the moment that a point of perfect foresight exists, and that the system is at that point. At this point, each agent knows the behavior of the other, and this belief is consistent with what actually occurs. Now, the problem is in part a conceptual one. If the system is at a point of perfect foresight, it cannot be anywhere else. Therefore, it is irrelevant to conjecture about how the other agent would respond to a change in one's tariff, since no other tariff can possibly exist. Further, it is not methodologically legitimate to differentiate T_1 with respect to T_2, since country one initially assumes that T_2 is endogenous.

In Bresnahan's notation, there may be an epsilon neighborhood about the Nash equilibrium, but the solution can be only at the Nash equilibrium. To demonstrate this claim, consider a simple application of proof by contradiction. First, for perfect foresight to exist, both countries must know what tariff the other will impose. Begin with both countries in equilibrium. Now, assume that country one imposes a different tariff. This tariff creates a contradiction since country two, possessing perfect foresight, believes that one's tariff will be the original tariff. As a result, it is irrelevant to consider what would happen if one country changed its tariff, since it is beyond its power to do so.

Although it is still possible for each country to possess a belief about how the other might react to a change in their tariff, there is no way to compare this belief to the "true" response, since there is no "true" response. Basically, if analysis of a perfect foresight case is possible, it must be conducted only at an equilibrium point. It would be possible to consider changes in the underlying parameters, either in comparative statics, or over time, but in general, these changes will not aid in the understanding of the reaction of one country to another's tariff, especially since both tariff rates are functions of all the parameters.

Before examining a method to generate perfect foresight solutions, it is necessary to consider some general characteristics of perfect foresight. The most important point is that there can be an infinite number of such solutions. That is, consider a Stackelberg leader/follower situation in which the leader country offers two choices to country two. One is a prohibitive tariff (by one) with no trade--which will be imposed if country two imposes any sort of tariff. The second offer is for country one to impose a tariff slightly less than prohibitive, and no tariff by country two. From the perspective of either country, the latter offer is better than the first. If two agrees to this trade, _a fortiori_, perfect foresight exists. Clearly, any number of such points could exist. Of course, this type of process does not show where such offers originate, nor does it provide a means for determining the optimal choice for country one to offer. The important point is that there can be an infinite number of perfect foresight solutions, and there does not have to be a relationship between them. In particular, there does not have to be a continuous relationship between tariffs in perfect foresight. (In fact, continuity is extremely unlikely.) That is, given the proper assumptions, any tariff combination could represent a perfect foresight solution. Hence, the objective is transformed into a search for an optimization process that will generate solutions that also represent perfect foresight.

An approach that shows more promise than that used by Bresnahan (1981) is to start from a point of imperfect information. That is, consider that each country attempts to maximize a Social Welfare function. For country one, this function is dependent on their own tariff, and what they believe will be country two's tariff. In a dynamic sense, one's belief could be a function of the tariff levels that country two imposed in previous periods. Country two could have similar beliefs with respect to country one's tariffs. Unlike the Cournot model, where the belief is held regardless of the outcome in the next round, the beliefs could be revised with each successive round. The result is that each country will possess a function that they believe describes the reaction of the other country. If this process of adaptive expectations continues long enough, and the process is stable, then eventually, each country should learn the "true" reaction of each country. That is, at some point, the beliefs about the reaction functions will remain the same, and will be accurate.

Finally, consider a similar process where country one believes that country two will retaliate in a manner that is dependent on the tariffs imposed by both countries in the last period. Now, consider what happens when both countries possess perfect foresight. That is, country one knows exactly what tariff country two will impose because they each know that the other maximizes welfare subject to a belief function. Hence, the objective is to find belief functions that are consistent in perfect foresight. This search is the foundation of the next two chapters.

CHAPTER 3. PERFECT FORESIGHT FOR TWO COUNTRIES

Introduction

In order to fully understand the retaliation process, it is necessary to explicitly include time in the model. That is, a model of foresight must express what beliefs each country holds at a given period in time. The variable (t) will be an index of time. Since each country is economically rational, they will both be attempting to maximize some measure of welfare from trade. Since the process will be modelled over time, each country is attempting to maximize the discounted present value of their welfare measure. The importing country will be called country one, and will impose a tariff $T_1(t)$ at time (t). Country two will be the exporting country, and will impose tariff $T_2(t)$ at time (t).

Consider the maximization problem from the perspective of country one. Note that in any time period (t), the welfare that country one gains from trade is a function of both tariffs: $T_1(t)$ and $T_2(t)$. Since $T_1(t)$ is under the direct control of country one, it is called the control variable. Observe that at time (t), from the standpoint of country one, $T_2(t)$ has already been imposed, and is exogenous. Hence, it defines the state of the system. That is, once $T_2(t)$ is set, it imposes limits on the welfare values that country one can achieve. For this reason, $T_2(t)$ is called the state variable.

If T_2 is fixed for all time periods, the problem reduces to a simple optimization over time with respect to the single control variable. Of course, this type of problem is just another manifestation of Cournot behavior. Since the goal is to introduce foresight, country one must have a belief about how country two will impose its next tariff. That is, country one believes that the next period tariff by two ($T_2(t+1)$) is a function of current tariffs $T_1(t)$ and $T_2(t)$. For the moment, it is not important where this belief comes from. The objective is to maximize the discounted sum of welfare, subject to this belief. Since there is no artificial limit to the number of time periods, there is an infinite time horizon. Therefore, it is necessary to consider solutions that are in steady state equilibrium. That is, equilibrium exists when each country imposes the same tariff in the current period (t) that it imposed in the last period. Fortunately, there is a technique for solving maximization problems of this type. The problem precisely fits the conditions of a dynamic programming model. For a given belief function (g), the maximization process will yield an optimal tariff for country one to impose that is a function solely of $T_2(t)$. Since the belief function expresses $T_2(t)$ as a function of last period's tariffs, the optimal tariff can be written as a function of the two last period tariffs.

The key is to observe that a similar maximization process is carried out by country two. That is, they maximize discounted producer surplus, subject to a belief about how country one's next period tariff is affected by the current tariffs. This process also generates an optimal tariff for country two that is a function of the tariffs imposed by both countries in the last period.

Summarizing, country one starts with a belief about two's tariff

$$T_2(t+1) = g[T_1(t), T_2(t)] \tag{3.1}$$

which generates an optimal tariff as a function of the last period tariffs

$$T_1^*(t) = h^*[T_1(t-1), T_2(t-1)] \tag{3.2}$$

Likewise, two starts with a belief about one's tariff

$$T_1(t+1) = h[T_1(t),T_2(t)] \tag{3.3}$$

and derives an optimal tariff

$$T_2{*}(t) = g{*}[T_1(t-1),T_2(t-1)] \tag{3.4}$$

The important point is that if $h \equiv h^*$ and $g \equiv g^*$, then the belief functions are equivalent to the "actual" responses, and perfect foresight exists, and the problem is solved. Hence, in order to generate perfect foresight solutions, it is necessary to find two functions (g and h) such that these relationships hold. Essentially, it is just a matter of solving a system of first order difference equations.

A good treatment of the dynamic control theory which underlies the solution presented here can be found in Kamien and Schwartz (1981). Selten and Marschak (1978) also commented on more general conditions of such models, but their analysis rests on "kinked" reaction curves. Radner (1980) examined naive cartel participants using trigger strategies in a finite game.

In the particular linear solution considered, there are five solutions which can be classified according to the reaction of one country to a change in tariffs by the second. First, the Cournot solution appears, in which neither country reacts to a change in the other's tariff. Second, there is a full retaliation solution, where any change in a tariff by one country is met with an equal change by the other country. There are two sub-cases to this solution, one set of tariffs is lower than the Cournot case (which will be a subsidy if the discount factor is greater than one-half), and one set is higher. The final solution arises from a reaction in which an increase in a tariff is met by an equal decrease in the other country's tariff. Again, there are two sub-cases, with one tariff lower than Cournot, and one higher.

Description of Model

Mathematically, from the viewpoint of country one, the process can be reduced to two equations. First, there is one's belief function

$$T_2(t+1) = g[T_1(t), T_2(t)] \tag{3.5}$$

Secondly, dynamic programming results show that the discounted present value of welfare can be written in two parts as

$$K[T_2(t)] = (U_1[T_1^*(t),\ T_2(t)] + \delta K(T_2(t+1))) \tag{3.6}$$

In equation (3.6), U_1 represents some measure of utility accruing to country one in time period t, and δ is a discount factor. The first term is the welfare accruing to the current period, and the second term is an iterative way to denote the present value of all future states. The right hand side of (3.6) is maximized with respect to the control variable ($T_1(t)$). Hence, K is determined solely by the state variable $T_2(t)$. In economic terms, K represents the maximum present value of any possible state. Observe that the problem has now been expressed as a dynamic programming problem with an infinite horizon.

Before continuing, it should be noted that some minor restrictions are necessary to justify equation (3.6). First, the utility function must by bounded for finite tariffs. Second, there must be some combination of tariffs that generates zero utility. If the problem is expressed to consider utility from trade alone, these two restrictions are fairly easily met, with no-trade fulfilling the second restriction. Finally, it must be possible to reach the state ($T_2(t)$) that generates no-trade in a finite number of steps. This last restriction imposes some minor constraints on the reaction belief expressed in equation (3.5), but economically, they do not matter, since the problem only makes sense if no-trade is allowed as a possible solution. In general, for a given belief, this system can be solved to find a reaction function for country one.

In the general problem, it is possible to show that the Cournot solution is also a perfect foresight solution. To prove this point, it is necessary to show that if both countries believe that the other will not change its tariff, then there is some point in tariff space at which it is true that neither country changes its tariff. Recall that, in general terms, one's initial belief about two (denoted g) led to an optimal tariff for one. The process could be written functionally as

$$T_1^*(t) = g\{f[T_1(t-1),T_2(t-1)]\} \qquad (3.7)$$

To see that the Cournot solution is possible, let country one assume that two follows a Cournot reaction, that is,

$$dT_2(t)/dT_1(t-1) = 0 = dT_2(t)/dT_2(t-1) \qquad (3.8)$$

In other words, country one believes that country two's tariff is a constant value that does not depend on the tariffs imposed in the last period. Using the first equality, will cause

$$dT_1^*(t)/dT_1(t-1) = 0 \qquad (3.9)$$

by the composite function rule and equation (3.7). Now, if country two possesses perfect foresight, they will know equation (3.9) is true, which, by the composite function rule again, will lead to their optimal tariff rule such that

$$dT_2^*(t)/dT_1(t-1) = 0 \qquad (3.10)$$

Since this equation represents two's true reaction, country one's initial belief was correct. A similar argument shows that $dT_2(t)/dT_2(t-1) = 0$ is also a perfect foresight solution. Hence, the Cournot solution is also a perfect foresight solution. Economically, it means that at that point, neither country expects the other country to change its tariff, and neither country does move.

It is important to note that the Cournot solution is not always a perfect foresight solution. The above case is special in that the utility function is dependent on the current period tariffs only. For example, if adjustment costs are added, the utility function will include a $T_1(t-1)$ variable. Consequently, the value function K must now include the same term, and the optimal tariff $T_1^*(t)$ will be a function of $T_1(t-1)$ both directly and indirectly (by composite function). Hence, the above perfect foresight analysis will fail because the true reactions will not be zero.

Note that the way the problem is posed, the only interesting solutions occur when the system is in steady state equilibrium. It might be possible to consider other solutions, however, these solutions would be artificial in that they would depend heavily on the external conditions imposed to generate them. For example, the game could be posed within a finite time period, but the solutions would depend on the starting point, and the length of time; or on some other ending criterion. Whether or not any of the original solutions remain would depend on whether or not the new conditions exclude them.

Specific Solution

To show that the procedure posed above has meaningful solutions, consider a more specific model. In the following example, country one shall be assumed to be exporting some good, and imposing a specific export tariff ($T_1(t)$) on some good, while country two imposes a specific import tariff ($T_2(t)$) on the same good. The simplifying assumption of linear supply and demand curves is added to the conventional assumption of pure competition in the domestic markets. Additionally, both countries have the same bargaining power. That is, each country will equally bear the burden of an increase in tariffs (in terms of prices). In particular, this assumption means that the elasticity of two's export supply curve must equal the elasticity of one's import demand curve. This assumption may seem unreasonably restrictive, but it is actually the most relevant case. That is, the process could be modelled with different elasticities, but then the results would seem to depend more on which country is less able to respond to tariff pressures. Hence, to remove extraneous considerations, this paper consider two countries that are equals. When dealing with linear excess supply and demand curves, this assumption has the effect of imposing symmetry in the two maximization processes that must be undertaken by the two countries. Net producer surplus and net consumer surplus, along with the respective tariff revenues (which are assumed to be redistributed as income), are taken as the approximation to utility for countries one and two respectively. Expressing these assumptions algebraically,

$$Qs = a_1+b_1P \tag{3.11}$$

$$Qd = a_2-b_1(P+T_1(t)+T_2(t)) \tag{3.12}$$

Equations (3.11) and (3.12) can be solved simultaneously for equilibrium price and quantity as functions of $T_1(t)$

and $T_2(t)$. The graphics of the problem are presented in figure 2.1. As shown in Appendix A, with linear demand and supply curves, the utility approximations are quadratic functions in $T_1(t)$ and $T_2(t)$. Hence, equation (3.6) can be expressed as

$$\begin{aligned} K[T_2(t)] = \max\{&[a+bT_1(t)+cT_2(t)+gT_1(t)T_2(t) \\ &+ eT_1(t)^2 + fT_2(t)^2] \\ &+ \delta K[T_2(t+1)]\} \end{aligned} \tag{3.13}$$

Equation (3.5) representing one's belief about two's reaction will be linear, and can be written as

$$T_2(t+1) = \mu + \alpha T_1(t) + \beta T_2(t) \tag{3.14}$$

The details of the solution are left to Appendix A, following the procedure outlined above. Note that the symmetry assumption implies that the actual reaction functions of each country must be symmetric with respect to $T_1(t)$ and $T_2(t)$, so that once the coefficients are found for country one, the symmetry condition automatically generates the coefficients for country two's reaction function. The linearity conditions create a quadratic description of the value function K, such that

$$K[T_2(t)] = B_0T_2(t) + B_1T_2(t)^2 \tag{3.15}$$

Now, It can be shown that the optimal value of $T_1(t)$ is given by

$$T_1(t)^* = (-H-S\mu)/Y - \alpha(S/Y)T_1(t-1)$$
$$- \beta(S/Y)T_2(t-1) \tag{3.16}$$

Observe that the optimal tariff is indeed a function of both last period tariffs. H, S, Y are substituted for convenience and are defined as

$$H = b + \mu\delta B_0 + 2\alpha\delta\mu B_1 \tag{3.17}$$

$$S = g + 2\alpha\delta\beta B_1 \tag{3.18}$$

$$Y = 2e + 2\alpha^2\delta B_1 \tag{3.19}$$

The basic results can be summarized in three cases, grouped according to the slope of the reaction function. There is the Cournot case with no retaliation; a full retaliation case, where an increase in a tariff is matched by an equal increase by the other country; and an opposite retaliation case, where an increase in a tariff is met with a decrease by the other country. First, consider the Cournot case, given by setting $\alpha = \beta = 0$:

$$B_1 = b_1/6 \tag{3.20}$$

$$B_0 = -(a_1+a_2)/3 \tag{3.21}$$

$$\alpha = \beta = 0 \tag{3.22}$$

$$\mu = (a_1+a_2)/2b_1 \ (1/2) \tag{3.23}$$

$$T_1^* = T_2^* = \mu \tag{3.24}$$

Equation (3.22) means that country two imposes a constant tariff (equal to μ) that does not depend on the tariffs in the last period. The second case is generated by setting $\alpha = \beta$, and $S = -Y$, and results in the following values:

$$B_1 = b_1/2 \qquad (3.25)$$

$$B_0 = -(a_1+a_2)/2 \qquad (3.26)$$

$$\alpha = \beta = \pm \sqrt{1/2\delta} \qquad (3.27)$$

$$\mu = (a_1+a_2)/2b_1 \ (1-\alpha) \qquad (3.28)$$

The steady state tariffs are

$$T_1{*} = T_2{*} = \mu/(1-2\alpha) \qquad (3.29)$$

Note that the main reaction ($\alpha = \beta$) means that if country one increases its tariff, country two will retaliate by increasing its tariff in the next period. Observe that the optimal tariffs for both countries are equal (because of the symmetry imposed above). Also, the tariffs are dependent on the constant term in the belief function (μ), and the reaction by the other country (α). The final case, generated by $\alpha = -\beta$ and $S = Y$ results in

$$B_1 = b_1/2 \qquad (3.30)$$

$$B_0 = -(a_1+a_2)/2 \ (1/(1+\alpha\delta)) \qquad (3.31)$$

$$\alpha = -\beta = \pm \sqrt{1/4\delta} \qquad (3.32)$$

$$\mu = (a_1+a_2)/2b_1 \ (1/2(1+\alpha\delta)) \qquad (3.33)$$

$$T_1{*} = T_2{*} = \mu \qquad (3.34)$$

In this case, as shown by (3.32), if country one increases its tariff, country two retaliates by decreasing its tariff in the next period. Once again, the optimal tariffs, from equations (3.33) and (3.34), depend on the reaction of the "other" country.

Before evaluating the results on a case-by-case basis, two additional points need to be considered. The first question is whether or not the results are maximum points and not minimum ones. Maximizing the right hand side of equation (3.6) with respect to $T_1(t)$ generated the optimal tariff by solving

$$\begin{aligned} 0 &= b + gT_2(t) + 2eT_1(t) \\ &\quad + \alpha\delta\{B_0 + 2B_1[\mu + \alpha T_1(t) + \beta T_2(t)]\} \end{aligned} \tag{3.35}$$

The second derivative of this equation with respect to $T_1(t)$ yields a second order condition (SOC) such that

$$SOC = 2e + 2\alpha^2\delta B_1$$

or, substituting in the proper values

$$SOC = b_1(\alpha^2\delta - 3/4), \tag{3.36}$$

which is negative for all the cases, so the points are indeed maximum values.

The second question to consider is whether or not a given solution is stable. That is, the solutions for the optimal tariffs as outlined in equation (3.16) represent a system of two linear difference equations.

The steady state solution of them is important, but it is also constructive to ask whether or not the system would return to that particular value if it was perturbed by some amount. Writing the equations in their symmetric form yields

$$T_1(t)^* = \mu + \alpha T_1(t-1) + \beta T_2(t-1) \tag{3.37}$$

$$T_2(t)^* = \mu + \beta T_1(t-1) + \alpha T_2(t-1) \tag{3.38}$$

Solutions can be postulated to be in the form

$$T_1(t) = Am^t \tag{3.39}$$

$$T_2(t) = Bm^t \tag{3.40}$$

Using these two equations, and simplifying equations (3.37) and (3.38) by writing the homogenous system in matrix form yields

$$\begin{bmatrix} m-\beta & -\alpha \\ -\alpha & m-\beta \end{bmatrix} \begin{bmatrix} A \\ B \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{3.41}$$

In order for a non-trivial solution to exist, the coefficient matrix must be singular, hence, its determinant must be equal to zero. That is,

$$(m-\beta)^2 - \alpha^2 = 0 \tag{3.42}$$

The two roots to the characteristic equation are then

$$m_1 = \alpha + \beta \tag{3.43}$$

$$m_2 = -\alpha + \beta \tag{3.44}$$

For a solution to be stable, the absolute value of the roots must be less than one. The question of stability will be considered further under each of the cases.

Case 1: Cournot

Consider the easier case first. That is, the Cournot case has been covered fairly extensively in the literature, so all that needs to be done is to determine if there are any new twists introduced in the perfect foresight analysis. First, putting $\alpha = 0$ into equation (3.36) indicates that the solution is indeed a maximum. Second, from equations (3.43) and (3.44), both roots of the characteristic equation are equal to zero. Which means that the system is degenerate. The equilibrium quantity can be solved for

$$Qe = (a_1+a_2)/4 \tag{3.45}$$

Notice that none of the results for this case are dependent on the discount factor (δ). Also, observe that $dT_1*/dT_2 = 0$ in equilibrium. That is, there is no response to a change in either country's tariff. Of course, in equilibrium, there is also no incentive to change. Hence, it is fairly clear that perfect foresight does hold, in that neither country expects the other to alter its tariff, and neither country does change.

In order to compare the various cases, it is useful to examine the level of welfare in the current period.

Observe first that T_1 must equal T_2 in equilibrium (denoted T*); and using equation (3.12), welfare for either country can be expressed as

$$U = a + (b+c)T^* + (g+e+f)T^{*2} \qquad (3.46)$$

Using the definitions of the variables (a-g) yields

$$U = a - b_1/2\ T^{*2} \qquad (3.47)$$

For the Cournot case, it reduces easily to

$$U = [(a_1+a_2)^2/8b_1]\ (1 - 1/4) \qquad (3.48)$$

which is greater than zero.

The optimal tariffs--from equations (3.23) and (3.24)--are composed of two parts: a first term, and a multiplier coefficient (1/2). It is clear from equations (3.28) and (3.33) that the tariffs in the other cases can be written in a similar manner; differing only in the multiplier. These multipliers are graphed against the discount factor in Figure 3.1. In this case, the graph is a straight line, halfway between free trade and no trade. (It is halfway because of the equal elasticities.) Note that welfare also consists of two parts: a first term based on the constant term from the welfare function, and a multiplier coefficient (1-1/4). For later reference, the last term of this coefficient (1/4) is graphed against the discount factor in Figure 3.2. Since it is not dependent on δ, it is not very interesting, but it does provide a reference

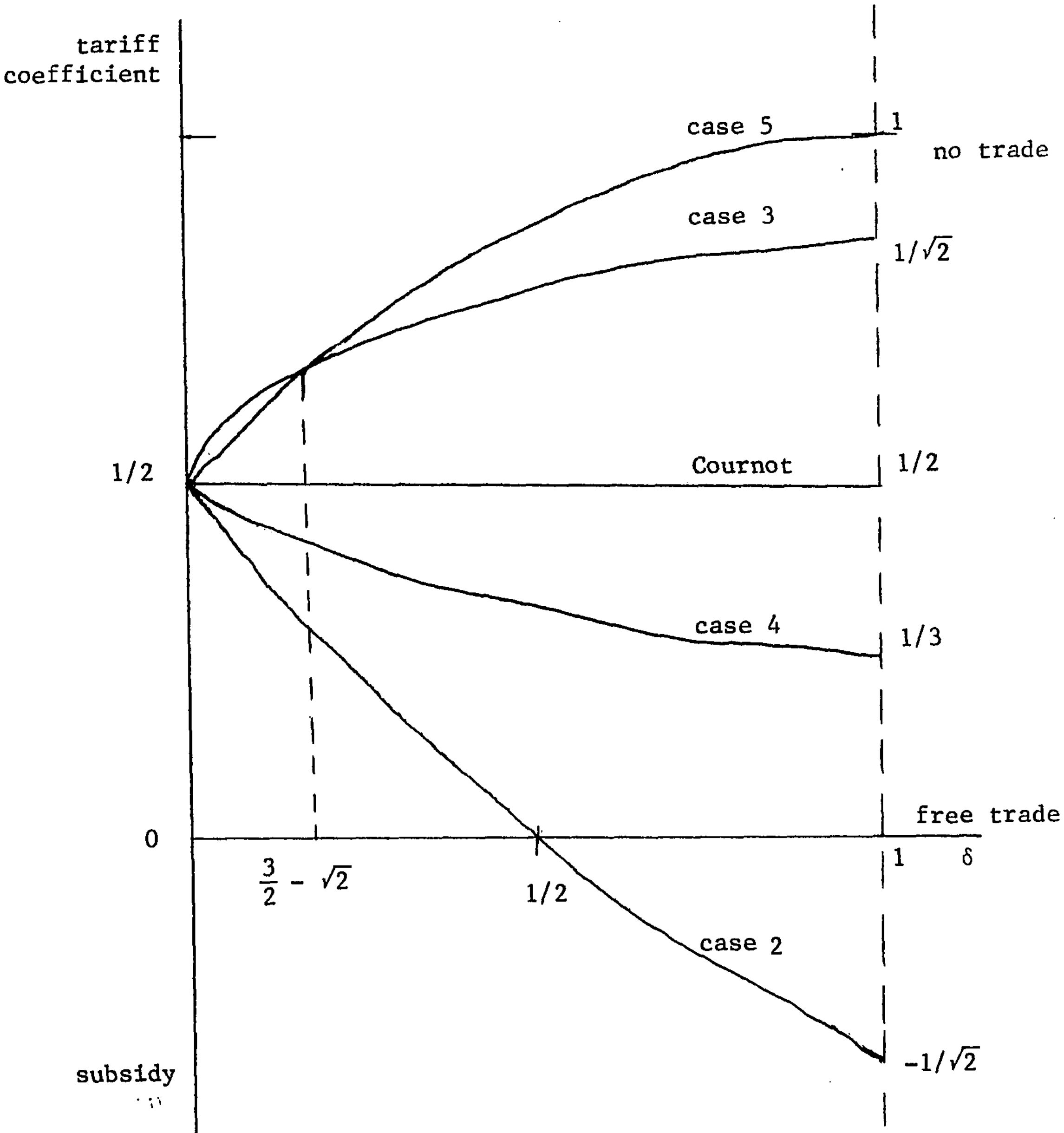

Figure 3.1
Comparison of tariffs

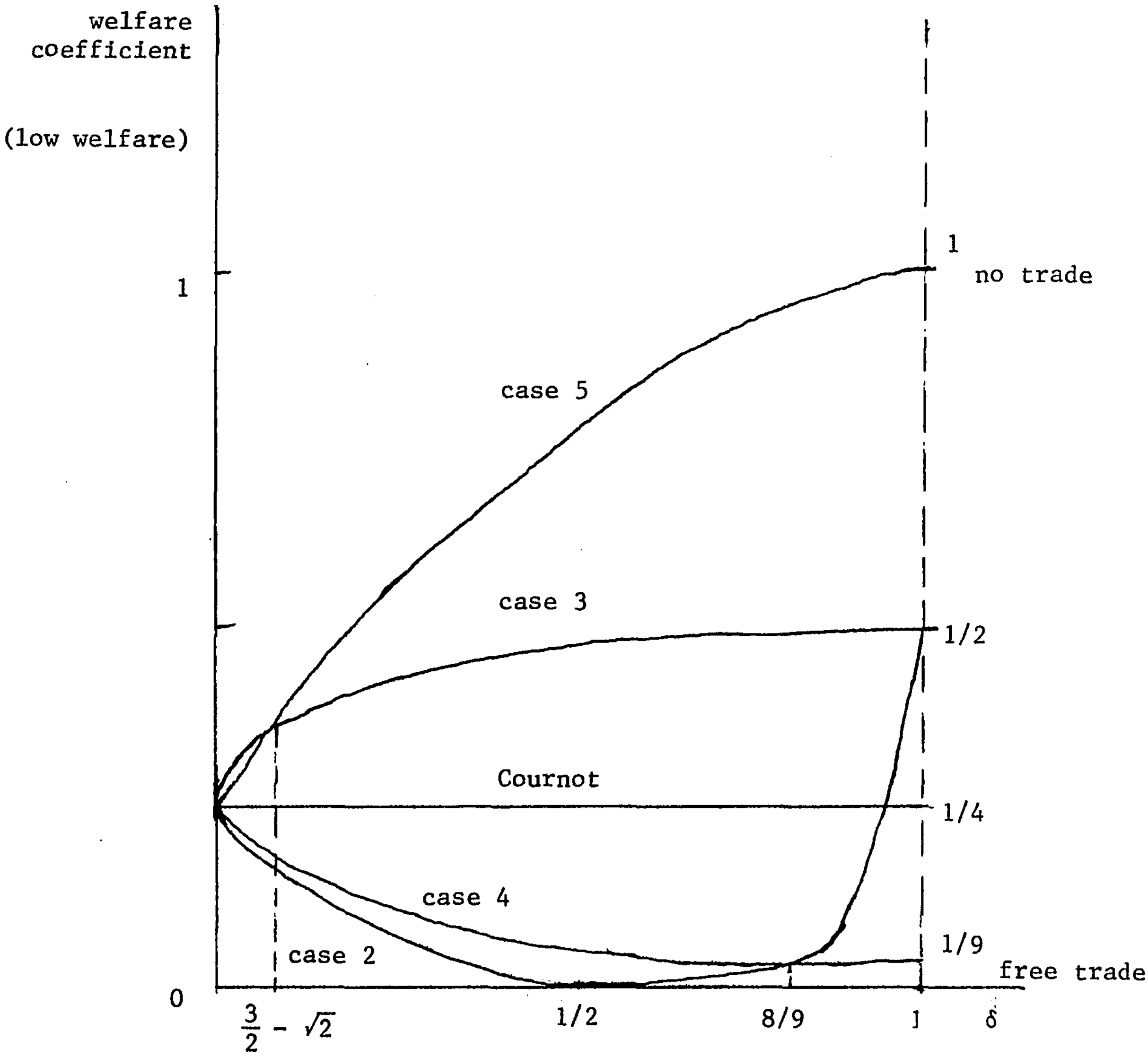

Figure 3.2
Comparison of welfare

for the other cases. It will be shown that welfare in the other cases can be written in a similar fashion. Note that a coefficient of one represents no trade, and zero is free trade.

Full Retaliation

In this case, where $\alpha = \beta$ and S=-Y, there are two sub-cases, created by the positive and negative square roots used in finding α and β. Since the optimal tariffs are dependent on α and β, it is necessary to consider both cases separately in most respects. However, for both cases, the second order condition is fulfilled--since it is dependent on the square of α. Also, by using the absolute value, it can be seen that, in each sub-case, one root is greater than one, and one is degenerate, implying that the solution is a saddle point, which means that if the initial belief is "correct," the solution will "converge" to this point.

<u>Case 2: subsidy sub-case</u>

Using the positive square roots as the first sub-case, the optimal tariffs from equation (3.29) are

$$T_1^* = \frac{(a_1+a_2)(\sqrt{2\delta} - 1)}{2b_1 \ (\sqrt{2\delta} - 2)} \qquad (3.49)$$

One way to understand the results is to examine the values for the tariffs (denoted T*) for differing values of the discount factor (δ). When δ is equal to one,

$$T^* = -(a_1+a_2)/(2b_1\sqrt{2}\ ) \tag{3.50}$$

From the equilibrium quantity under free trade, a_1+a_2 must be greater than zero. Since b_1 is the slope of the supply curve it is positive as well, so the optimal tariffs in this case are negative, or subsidies as they are usually called. For a discount factor equal to one-half, T* is equal to zero--which is free trade. In the limiting case, as the discount factor goes to zero, the optimal tariffs approach

$$T^* \rightarrow (a_1+a_2)/4b_1 \tag{3.51}$$

which is the equilibrium tariff level for the Cournot case. It is clear that as the discount factor decreases from one to zero, the optimal tariffs monotonically increase from a subsidy to no tariff to the Cournot tariff level. The coefficients demonstrating these values are displayed in figure 3.1.

Why does one-half yield the free-trade solution? The easiest way to answer the question is to split welfare into two categories: current welfare, and discounted future welfare. In steady state equilibrium, the actual welfare received in any period is a constant, so the discounted future value is merely an infinite geometric series. When the discount factor is one-half, the discounted sum is exactly equal to the welfare received in the current period. That is, when each country expects the other to retaliate in full, the optimal tariff will be zero if society places an equal weight on current welfare and future welfare. However, if society values future consumption more, then each country will employ subsidies to ensure that the other country will not impose a tariff in the future (diminishing the more important welfare). If society values current more than future welfare, then each country will impose a tariff now, in order to increase current welfare--even if it carries a cost of lower welfare in the future.

For this case, the steady state value of current welfare is given by

$$U = (a_1+a_2)^2/8b_1 \cdot (1 - (1-\alpha)^2/(1-2\alpha)^2) \qquad (3.52)$$

First note that this value is always positive. Secondly, to determine how welfare changes as the discount factor varies, differentiate (3.52) with respect to α. The result is

$$dU/d\alpha = -(a_1+a_2)^2/4b_1\ (1-\alpha)\ (1-2\alpha)^{-3} \qquad (3.53)$$

Since $d\alpha/d\delta$ is always negative, current welfare increases when the discount factor increases if the discount factor is between zero and one-half. If the discount factor is between one-half and one, and increasing, current welfare will decrease. Note that when the discount factor increases between one-half and one, the total welfare level increases, but welfare accruing from the current period decreases. These relationships are presented in figure 3.2, based on the coefficient multiplier (the right half of (3.52)). This multiplier first decreases to zero and then rises to one-half, which means that welfare increases to its free trade level, and then drops below the Cournot level.

Case 3: tariff sub-case

The second sub-case is very similar to the first sub-case. All of the formulas are the same, but it uses the negative roots for α and β. As a result, the optimal tariffs are

$$T^* = \frac{(a_1+a_2)(\sqrt{2\delta}+1)}{2b_1\ (\sqrt{2\delta}+2)} \tag{3.54}$$

When the discount factor is one,

$$T^* = (a_1+a_2)/2b_1\sqrt{2} \tag{3.55}$$

As the discount factor approaches zero, in the limit,

$$T^* \rightarrow (a_1+a_2)/4b_1 \tag{3.56}$$

Note that equation (3.56) is the Cournot equilibrium level again.

In this second sub-case, as society's value of future consumption decreases; the discount factor goes from one to zero; the optimal tariff monotonically decreases from a finite level (trade still exists) to the Cournot tariff. Observe that the current welfare in this case is also given by equation (3.52), and it is always positive. In order to examine the change in current welfare with respect to changes in the discount factor, recall equation (3.53). Since $d\alpha/d\delta$ is now positive, the sign of $dK/d\delta$ is now negative. Which means that as the discount factor increases, the steady state level of tariffs increases, causing the current welfare to decrease.

The primary difference between the sub-cases is the response of one country to its tariff in the last period. In the first case, if either country increased its tariff in the last period, both countries will increase their tariff in the current period, and there will be a continuous increase in the tariffs. In the second sub-case, an oscillating pattern will develop, because if country one increased its tariff in period one, both countries will decrease their tariff in period two; and increase them in period three, etcetera.

Opposite Retaliation

In the last case, where $\alpha = -\beta$ and $S = Y$, there are also two sub-cases, created by the positive and negative square roots in finding α and β. Note that the second order condition is fulfilled for both sub-cases, since it depends on the square of α. Also, the system is a saddle point, since one root is zero, and the absolute value of the other is greater than one.

<u>Case 4</u>

The steady state tariff in this case is

$$T_1^* = T_2^* = (a_1+a_2)/2b_1 \ 1/2(1+\alpha\delta) \qquad (3.57)$$

Note that this value is always positive. Further, when the discount factor equals one,

$$T^* = (a_1+a_2)/2b_1 \ (1/3) \qquad (3.58)$$

Once again, as the discount factor approaches zero, the optimal tariff monotonically approaches the Cournot tariff level. Notice in Figure 3.1 that this tariff is between the Cournot tariff and the full retaliation (case 2) tariff, but it is always a tariff (never a subsidy).

The current level of utility is found by

$$U = (a_1+a_2)^2/8b_1 * 1 - 1/4(1+\alpha\delta)^2) \qquad (3.59)$$

It is clear that as the discount factor increases, the optimal tariff decreases, and the level of welfare increases. Note in figure 3.2 that when the case 2 tariff becomes a subsidy, the corresponding welfare drops, until, for values of the discount factor close to one, case 4 has the highest level of welfare.

Case 5

The formulas for this sub-case are very similar to those in the last one, since the only difference is the sign of the α term. Once again, as the discount factor approaches zero, the optimal tariffs approach the Cournot level. However, as the discount factor approaches one, the tariffs monotonically increase until they reach the no-trade level when the discount factor is equal to one (the discount rate is zero). Of course, as the tariffs increase, the welfare (from trade) decreases to zero. As shown in figures 3.2 and 3.2, this case clearly has the highest tariffs, and consequently, the lowest level of welfare.

Comparison of Cases

For this model, there are now three classes of perfect foresight solutions. Since two of them have two sub-cases, there are five possible solutions. In order to compare the solutions, it is necessary to observe the values of the tariffs for different values of the discount factor. Note that all of the tariff formulas have a common structure--a common constant term that is multiplied by some fraction. This fraction is dependent only on the discount factor in all the cases. The current welfare in the cases can be written in a similar manner, where the multiplier is one minus the square of the tariff multiplier. The multipliers for the tariffs are plotted against the discount factor in figure 3.1, and the current welfare multipliers are displayed in figure 3.2, where a lower line represents a higher tariff. Note that the values all converge to the Cournot case when the discount factor is zero (the future has no value). The tariffs are essentially ranked from low to high as follows: full retaliation (subsidy sub-case); opposite retaliation (low tariff); Cournot; full retaliation (tariff); opposite retaliation (high tariff). The two highest tariffs cross when the discount factor equals $(3/2 - \sqrt{2})$; as do the corresponding welfare values. The subsidy-case welfare crosses the Cournot level when the discount factor equals (8/9). It crosses the opposite retaliation (low tariff) welfare when the discount factor solves

$$0 = 2\delta^2 - (19+2\sqrt{2})\delta + 16 \qquad (3.60)$$

It should be noted that the ranking of the welfares must be approached with caution. That is, a country does not really choose which solution should exist based on which one carries higher welfare. There are five solutions to the problem that are consistent with perfect foresight. Each one arises from a different set of beliefs about how both countries will react to changes in tariffs. Each one is independent of the other solution, in that the countries cannot "move" from one solution to another. Any movement from an equilibrium position results in an unstable situation, and imperfect foresight. The ranking of the welfares is relevant only because it shows which set of beliefs is the most beneficial. For example, if the prevailing solution was Cournot, and a country wanted to increase its welfare by moving to the full retaliation case, the only way to get there is to change the beliefs of both countries about how the other will react.

Since the Cournot case has already been described, consider the full retaliation case. Note that there are actually two reactions of interest in the model. The reaction by the other country to a tariff change, and the response by the first country to its own tariff change in the last period. In this case, both reactions are in the same direction. In the first sub-case, if a country raises its tariff, both effects will cause the two countries to match the other's tariff increase indefinitely. Hence, at equilibrium, neither country has an incentive to change the tariff, for fear of the ultimate retaliation. In the second sub-case, both tariffs will again move together (up in the same time period, or down in the same time period). However, since a country's reaction to the foreign change is negative, it is not fear of retaliation that causes an equilibrium point. Rather, a country knows that if it increases its tariff, the other country will merely decrease its tariff by the same amount to negate the change; hence, eliminating any possible gain in consumer or producer surplus. Possibly the reason this equilibrium tariff level is higher than the first is because the country with the higher tariff will get more tariff revenue, and tariff revenue is now a more significant factor since changes in consumer and producer surplus are negated. The reaction of a country to changes in its tariff from the previous period causes the tariff level (out of equilibrium) to oscillate.

This oscillation seems to provide a moderating influence on the equilibrium tariff level.

In the opposite retaliation case, in a given time period, the two tariffs are always moving in opposite directions. For the first sub-case, a response to a change in the other country's tariff is in the same direction. That is, there is the fear of retaliation again. Note that since the response to changes in one's own tariff is negative, an oscillation pattern develops again--distorting the discounted welfare sum, and causing a slightly higher tariff than the first case of the full retaliation. This oscillation is removed in the second sub-case, and the opposite retaliation is much clearer. Here, if a country raises its tariff, the other will decrease its tariff, and this process will continue indefinitely. In this sub-case, if society's social discount rate is zero, the equilibrium tariffs are high enough to completely eliminate trade.

Conclusion

The primary focus of this chapter has been to show that it is possible to find an optimal tariff for which both countries possess perfect foresight. The Cournot equilibrium represents perfect foresight since both countries believe that neither will change their tariff. Two solutions also arise when both countries believe that the other will exactly match a tariff increase or decrease. Basically a "strong-arm" situation. The final solutions come about when both countries believe that any tariff increase will be met with an equal decrease by the other country--negating the change, and merely redistributing the tariff revenue. Depending on the discount factor involved, the full retaliation case can lead to free trade (or even a subsidy); and the opposite retaliation can lead to a no-trade solution.

CHAPTER 4. M EXPORTERS AND N IMPORTERS

One of the drawbacks of the optimal tariff with perfect foresight analysis presented in Chapter 3 is that to solve the system, both countries had to have equal economic power against the other. That is, the absolute values of the supply and demand curves had to be equal. Because of the nature of the equations, it is not possible to relax this assumption without introducing an infinite number of solutions. However, it is possible to introduce more than two countries into the analysis. Any one country will have the same economic power (slope of supply or demand curve) against any other single country. However, if there is more than one exporter (m in general terms) or more than one importer (n in general terms), the economic position of any one exporter or importer has now changed. Hence, it is possible to consider what happens in a market as one country loses its market share.

Introduction

In general terms, the problem is solved by noting that from the perspective of an importing country, there are three classes of countries. Country A is one importing country, group B is the other (n-1) importing countries, and group C consists of the (m) exporting countries. Since all countries in a particular class behave the same, the problem is very similar to the one posed in chapter 3. Country A attempts to maximize the discounted present value of a welfare measure, subject to a belief about how the other two types of countries will respond. This belief can be denoted by expressing the next period tariffs by the other two countries (B and C) as function of the current period tariffs imposed by B and C. Solving this maximization problem in an infinite time horizon leads to an optimal tariff for country A that is a function of the last period tariffs. The other two types of countries use a similar process to generate their optimal tariffs as functions of the last period tariffs. Now, for perfect foresight to exist, all countries must know the "true" coefficients of these various response functions. Since all countries know how the others make their decisions, the coefficients can be found by equating the original "estimates" with the "true" values. This process yields of system of equations that can be solved, yielding equilibrium tariffs.

Since any one country must have the same individual power as any other, the slope of any excess supply or demand curve must be of the same magnitude--call it b_1. Initially, consider the problem from the perspective of one importer (country A). The objective is to maximize some measure of welfare. Here, consumers' surplus and redistributed tariff revenue will suffice as approximations. Price (P) is expressed as world price, and the demand for imports within country A can be expressed in linear terms as

$$Qa = a_1 - b_1(P+Ta) \tag{4.1}$$

All of the other (n-1) importers are assumed to be identical to this country, so their total demand can be written

$$Qb = (n-1)a_1 - (n-1)b_1(P+Tb) \tag{4.2}$$

Finally, all of the (m) exporters are identical, so the amount they are willing to supply can be expressed as

$$Qc = (m)a_2 + (m)b_1(P-Tc) \tag{4.3}$$

The details of the mathematics are left to Appendix B, but by adding equations (4.1) and (4.2) to get world demand, and equating it to the world supply equation (4.3), it is possible to solve for equilibrium world price. Once world price is found, equation (4.1) yields equilibrium quantity demanded by country A:

$$Qa = m(a_1+a_2)/(m+n) + b_1/(m+n) \cdot [(1-m-n)Ta + (n-1)Tb - mTc] \qquad (4.4)$$

The welfare approximation can be written in terms of equilibrium quantity, and the expression in equation (4.4) can be substituted to generate an expression for welfare that is a quadratic function of the three tariff rates (Ta, Tb, Tc):

$$\begin{aligned} Ua = {} & [m/(m+n)]^2 \\ & + [m(m+n)-2mb_1(m+n-1)]\ (a_1+a_2)/(m+n)^2\ Ta \\ & + 2mb_1(n-1)(a_1+a_2)/(m+n)^2\ Tb \\ & - 2m^2b_1(a_1+a_2)/(m+n)^2\ Tc \\ & + (n-1)b_1/(m+n)^2\ TaTb \\ & - mb_1/(m+n)^2\ TaTc \\ & - m(n-1)b_1/(m+n)^2\ TbTc \\ & - (m+n-1)(m+n+1)b_1/2(m+n)^2\ Ta^2 \\ & + (n-1)^2b_1/2(m+n)^2\ Tb^2 \\ & + m^2b_1/2(m+n)^2\ Tc^2 \end{aligned} \qquad (4.5)$$

To simplify notation, it can be written as

$$Ua = a + bTa + cTb + dTc + eTaTb + fTaTc + gTbTc + hTa^2 + iTb^2 + jTc^2 \qquad (4.6)$$

It is also possible to go through a similar process for an exporting country. Because the same value of b_1 is used as the slope of the excess supply curve, the results are symmetric to equation (4.5). Which means that once results have been found for one type of country, the results for the other class follow immediately by symmetry. This symmetry is not only convenient, but necessary. Without the symmetry, there is an infinite number of solutions, which means that the analysis could provide no substantive conclusions.

The framework of the solution is a basic dynamic programming model. That is, country A is attempting to maximize the welfare function presented in equation (4.6), but it is necessary to consider the reactions of the other two countries. Hence, initially, country A believes that the future tariffs of the two countries can be expressed as

$$Tb_{t+1} = \mu + \alpha Ta_t + \beta Tc_t \qquad (4.7)$$

$$Tc_{t+1} = \varepsilon + \gamma Ta_t + \theta Tc_t \qquad (4.8)$$

For the moment, the reaction coefficients in these two equations are assumed to be known. Note that Tb_t does not appear explicitly in either equation. This omission is deliberate, since Tb_t appears implicitly through Ta_t. To include it again, would be superfluous, and would generate interdependent equations in the system.

To solve the system, it is necessary to find a function (K) that solves

$$K(Tb_t, Tc_t) = Ua(Ta_t^*, Tb_t, Tc_t) + \delta K(Tb_{t+1}, Tc_{t+1}) \qquad (4.9)$$

The function K represents the sum of welfare from the current period, plus welfare from future periods discounted by the factor (δ). Since the welfare function is quadratic, K will also be quadratic, and can be written

$$K(Tb_t, Tc_t) = BTb_t + CTc_t + DTb_t^2 + ETb_tTc_t + FTc_t^2 \qquad (4.10)$$

When no confusion will result, the (t) subscript will be dropped. The problem can now be solved by finding the values of the coefficients of K that will maximize the welfare function, subject to the reaction beliefs presented in equations (4.7) and (4.8).

The first step in solving the system is to differentiate (4.10) with respect to Ta_t, set the result equal to zero, and solve for the optimal Ta_t as a function of Tb_t and Tc_t. The values for the coefficients of K can be found by differentiating (4.10) with respect to Tb_t and Tc_t. The result is that if the reaction coefficients (from equations (4.7) and (4.8)) are known, the optimal Ta_{t+1} can be expressed as a function of Ta_t and Tc_t; or as a function of Tb_t and Tc_t. That is,

$$Ta_{t+1} = -(H+\mu e+zS)/Y - (\alpha e+\gamma S)/Y\ Ta_t$$
$$-(\beta e+\theta S)/Y\ Tc_t \qquad (4.11)$$

Or,

$$Ta_{t+1} = (\varepsilon-\theta H/Y) - (\theta e/Y)\ Tb_t$$
$$+ (-\theta S/Y + \gamma)\ Tc_t \qquad (4.12)$$

Where S, Y, and H have been defined for notational convenience as

$$S = f + \beta\delta(2\alpha D+\gamma E) + \theta\delta(\alpha E+2\gamma F) \qquad (4.13)$$

$$Y = 2h + \alpha\delta(2\alpha D+\gamma E) + \gamma\delta(\alpha E+2\gamma F) \qquad (4.14)$$

$$H = b + \delta(\alpha B+\gamma C) + \mu\delta(2\alpha D+\gamma E)$$
$$+ \varepsilon\delta(\alpha E+2\gamma F) \qquad (4.15)$$

Because of the symmetry noted above, these country A reaction functions are symmetric to the reaction functions presented in equations (4.7) and (4.8). Hence, if there is perfect foresight, the coefficients of these equations must by equal to the coefficients in equations (4.7) and (4.8) respectively. Which means that

$$\varepsilon = -(H + \mu e + \varepsilon S)/Y \tag{4.16}$$

$$\gamma = -(\beta e + \theta S)/Y \tag{4.17}$$

$$\theta = -(\alpha e + \gamma S)/Y \tag{4.18}$$

$$\mu = \varepsilon + \alpha H/e \tag{4.19}$$

$$\alpha = -\theta e/Y \tag{4.20}$$

$$\beta = -\theta S/Y + \gamma \tag{4.21}$$

There are now eleven nonlinear equations and eleven variables: B, C, D, E, F, μ, α, β, ε, γ, and θ. The system can be split into two sets. The first consists of the variables D, E, F, α, β, γ, and θ; and seven equations that contain only those variables. The second set consists of the other four variables. Given equilibrium values for variables in the first set, it is possible to analytically solve the second set of equations. Unfortunately, it is not possible to analytically solve the first set of equations. However, two important relationships can be derived from the first set:

$$\gamma^2 = (\theta + \alpha)^2 \tag{4.22}$$

$$\beta = \pm 2\theta \tag{4.23}$$

The plus/minus depends on the sign used in the square root from equation (4.22). Note that equation (4.22) is very similar to the results in chapter 3. In that case, the reaction by the exporter was equal to (or opposite of) the reaction by the importer. In this case; as perceived by an importer; the reaction by the exporter is equal to (or opposite of) the sum of the reactions by the two sets of importers. Second, it appears from equation (4.23) that the "other" importing countries will respond twice as much to a change in the exporting countries's tariffs. However, recall that Tb_t does not appear explicitly in equations (4.7) and (4.8), a fact which is taken into account by the two multiplier.

One of the important conclusions that can be reached from equations (4.16) to (4.20) is that the Cournot case is a solution to the system. It is also possible to examime what happens to individual solutions as the number of exporters (m) or importers (n) becomes infinite. Note that, including the Cournot solution, there are five possible solutions to the equations; as indicated by the squared terms in equation (4.22). To examine specific results in most of the cases, it is necessary to generate numerical solutions to the equations. However, the Cournot case is easier, so it is considered first.

Cournot

The Cournot case uses the assumption that a change in a tariff by any one country is met by no change in tariffs by the other countries. In terms of this model, it means that the four main reaction coefficients in equations (4.7) and (4.8) (α, β, γ, and θ) are all equal to zero. Substituting these values into the system equations yields the steady state level of tariff

$$T^* = \mu = -b/(e + f + 2h) \tag{4.24}$$

Because of the symmetry involved, any individual country will impose this same level of tariff, which reduces to

$$T^* = \frac{(m(m+n) - 2mb_1(m+n-1))\ (a_1+a_2)}{(m+n+1)(m+n)b_1} \tag{4.25}$$

Other than its historical significance, the reason the Cournot case is important is because it can be compared to the other cases. For the smallest case that can be considered, one exporter, and two importers, the optimal Cournot tariff is

$$T^* = (3-4b_1)(a_1+a_2)/(12b_1) \tag{4.26}$$

Since the slope (b_1) is positive, the sign of the tariff hinges on the magnitude of b_1. If the curves are relatively steep, then the tariff will be greater than zero. Otherwise, the optimal tariff will be negative (usually called a subsidy). As the number of importers increases (to infinity), the optimal Cournot tariff will approach zero. Which means that as an importing country becomes smaller and smaller relative to the world market, it loses control over price, and its ability to gain by changing prices is eliminated, so the country ultimately will impose no tariff or subsidy. This result is similar to the traditional statement that the optimal tariff for a small country is zero because it cannot influence the terms of trade.

From the viewpoint of the seller, it is important to note that the world price approaches the intercept value on the individual demand curves. That is, for an individual importing country, the price approaches its maximum level, and the level of imports approaches zero. Since there are a large number of buyers, the exporter can sell all it wants to at that price, so there is no incentive to grant a subsidy. Imposing a tariff is fruitless since price cannot go any higher, so the optimal value is no tariff.

On the other hand, holding the number of importers constant, and increasing the number of exporters, the optimal Cournot tariff approaches

$$T^* = (1-2b_1)(a_1+a_2)/b_1 \qquad (4.27)$$

Note that this tariff is non-zero. In fact, for most cases, it is a subsidy. To see why both exporters and importers would grant a subsidy, it is necessary to examine the world price level:

$$Pw = -a_2/b_1 + T^* \tag{4.28}$$

If there were no tariffs or subsidies, the prevailing price would be just the first term in equation (4.28), which is the lowest possible price on the exporters' supply schedules. That is, without international intervention, the few importing countries would pay the lowest possible price to obtain a minimal amount from any individual supplier. It is worthwhile for the exporting countries to grant subsidies on their exports. Also, the importing countries will find it beneficial to subsidize imports to increase the returns to consumers.

Numerical Solutions

As noted above, it does not seem possible to solve the system of equations analytically, hence, a numerical process was used to generate solutions for several different cases. The technique seemed especially relevant since individual solutions are not very important. It is much more meaningful to observe how the solutions--in terms of the reaction coefficients and the tariffs--change as the number of countries changes.

The above equations involving the reaction coefficients (α, β, γ, θ), and the coefficients of the state equation (D-F) were first reduced analytically as far as possible. The result is a reduction to the point where they behave as a system dependent on two variables: α and θ. That is, for given estimates of those two variables, two new estimates follow immediately from the equations. Once the first set of equations is solved, the values can be substituted into the analytical results for the second set. The problem is that the first set of reduced equations is very unstable numerically, in that poor guesses for the initial estimates can prevent the numerical system from converging. Therefore, to solve the system, it is necessary to use a successive bisection technique. Although it converges slowly, at least it converges, given "good" initial estimates. The second problem is that it is necessary to solve for two variables simultaneously.

The process of solving such a system can be outlined by examining two variables (X and Y). The basic computer algorithm starts with initial estimates for both variables. X_1 and X_2 are respectively the low and high values of X. Similarly, Y_1 and Y_2 are the high values of Y. The equations to be solved are expressed so that equilibrium values of X and Y will yield two zeros. That is, $f(X^*,Y^*) = (0,0)$. The initial values of X and Y have to be given such that $f(X_1,Y_i) \cdot f(X_2,Y_i) = (-a,b)$. That is, for both of the Y endpoints, the functional value corresponding to the X term must change signs between the two X points. Hence, there must be an X value between them such that $f(X^*,Y_i) = (0,b)$. This X value can be found by successively bisecting the interval, until the interval containing the zero is acceptably small. Further, the initial Y points have to have the property that $f(X_1^*,Y_1) \cdot f(X_2^*,Y_2) = (0,-b)$. That is, once the zeros for X have been found, the corresponding functional values for the Y term must change sign between the two values for Y. Hence, there will be a point between the two Y estimates such that $f(X^*,Y^*) = 0$. This point can be found by successively bisecting the Y interval. Of course, every time a new Y value is calculated, it is necessary to use the bisection technique to generate a new estimate for the X zero corresponding to that Y value. By decreasing the size of the final interval, the X* and Y* estimates can be as close to the actual values as desired. At least, up to the limit of the accuracy of the machine due to round-off errors. In most cases, the limit specified was five decimal places. Once the entire system was solved, the values were substitued back into the original equations to check the error. The results were also tested in sixteen digit precision in case round-off error was large, but there were no significant changes in the results. For the most part, the results appear accurate to four decimal places.

As pointed out above, because of the square terms, there are four basic solutions to the equations (not counting Cournot). One set of solutions is found for each value of the square root term found in calculating γ. Recall that γ is the response of the exporting countries to changes in the tariffs of the importing countries. The two cases within each set are found by choosing different starting values for the two intial estimates (α and θ). That is, for the first set, $\gamma = (\alpha+\theta)$ which is greater than zero. Hence, importing and exporting countries both increase their tariffs if the other type of country increased their tariff in the last period. For the second set, the same equality holds, but it is now less than zero. That is, if the importing countries raise their tariffs, the exporting countries will tend to lower their tariffs. The importing countries will respond in a similar fashion to changes in tariffs by the exporting countries. The other sign of the square root term is used to compute the third set, so $\gamma = -(\alpha+\theta)$ and it is greater than zero. In this case, an increase in tariffs by the importing countries results in a decrease in tariffs by the exporting countries, but an increase in tariffs by the exporting countries results in an increase in tariffs by the importing countries. The γ term is less than zero in the fourth set, and just the opposite reactions occur.

It is possible to analytically observe what happens to the reaction coefficients as more and more exporters and importers enter the market, and the number of buyers and sellers approaches infinity. As the number of importers (n) approaches infinity, all of the coefficients of the welfare function approach zero except

$$h = -b_1/2 \tag{4.29}$$

$$i = b_1/2 \tag{4.30}$$

Since (e) approaches zero; and (Y) does not; the reaction coefficient α must approach zero. That is, importers no longer respond to changes in tariffs by other importers. Hence,

$$\gamma^2 = \theta^2 \tag{4.31}$$

$$S^2 = Y^2 \tag{4.32}$$

In a similar fashion, holding the number of buyers constant, consider what happens as the number of exporters is increased. In this case, all the welfare coefficients are zero, except

$$a = (a_1 + a_2)^2/(2b_1) \tag{4.33}$$

$$b = -2b_1(a_1 + a_2) \tag{4.34}$$

$$d = -2b_1(a_1 + a_2) \tag{4.35}$$

$$h = -b_1/2 \tag{4.36}$$

$$j = b_1/2 \tag{4.37}$$

Before considering the numerical results for each of the cases, it is interesting to examine the reactions and tariffs in the limit as n and m approach infinity.

Holding the number of exporters constant, and increasing the number of importers, the following results follow fairly easily from observing that x approaches zero:

$$\gamma^2 = \theta^2 \tag{4.38}$$

$$\beta = \pm 2\theta \tag{4.39}$$

$$Y = -1 \tag{4.40}$$

$$S^2 = 1 \tag{4.41}$$

Using the definitions of Y and F, it is possible to show that

$$4\delta^2 b_1(\gamma^4) + \delta b_1(\gamma^2) + (1-b_1) = 0 \qquad (4.42)$$

Equation (4.42) can be used to solve for four values of γ. Two of these are imaginary, and the other two help create the four possible cases.

As the number of exporters increases, holding (n) constant, the values for p are solutions to

$$\gamma^2 = (b_1 - 1)/(2\delta b_1) \qquad (4.43)$$

As both (m) and (n) approach infinity, γ is determined from

$$\gamma^2 = (b_1 - 1)/(\delta b_1) \qquad (4.44)$$

To observe what happens to the optimal tariffs, it is necessary to determine how μ and ε react to changes in m and n. Note that as α approaches zero, μ approaches ε, and B and H approach zero. To solve for ε, the general procedure is to use the equations defining H and C. However, for the case where (n) approaches infinity, the two equations are not independent, so the tariff values cannot be found analytically.

On the other hand, it is possible to decide what occurs when the number of exporters increases. For the first two cases, where γ and θ are equal, the equations defining C and H can be reduced to

$$\varepsilon = -d/(2\delta\gamma F) \qquad (4.45)$$

Also, reducing the equation that defines the tariffs yields

$$T^* = \varepsilon/(1 - 2\gamma) \tag{4.46}$$

For the last two cases, where γ is the negative of θ, the equations reduce to

$$\varepsilon = -d(1 + 2\delta\gamma)/(2\delta\gamma F) \tag{4.47}$$

Note that in the last two cases, the sign of (ε) may depend on the magnitude of the discount factor. The optimal tariff is found by

$$T^* = -\varepsilon/(1 + 2\gamma) \tag{4.48}$$

Finally, note that as both (m) and (n) increase, d approaches zero, so ε must approach zero. That is, regardless of the particular case, as the number of importers and exporters increases, the optimal tariffs approach zero. Which means that as the number of participants in the market increases, the market approaches free trade.

Another important question that arises is whether or not the solutions are stable. That is, if the system is perturbed by some small amount, it would be nice to know if the system will return to any of the particular solutions, or merely diverge. In general terms, the three main reaction functions can be considered as a system of difference equations:

$$Ta_{t+1} = \mu + \alpha Tb_t + \beta Tc_t \tag{4.49}$$

$$Ta_{t+1} = \varepsilon + \theta Ta_t + \gamma Tc_t \tag{4.50}$$

$$Tc_{t+1} = \varepsilon + \gamma Ta_t + \theta Tc_t \tag{4.51}$$

To test for stability of the system, it is necessary to consider the solution to the homogeneous system.

Consider that the solutions (for the tariffs) must be of the form

$$Ta_t = Ak^t \tag{4.52}$$

$$Tb_t = Bk^t \tag{4.53}$$

$$Tc_t = Ck^t \tag{4.54}$$

Hence, the system can be written in matrix notation as

$$\begin{bmatrix} (\theta-k) & 0 & \gamma \\ -k & \alpha & \beta \\ \gamma & 0 & (\theta-k) \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{4.55}$$

For any non-trivial solutions to exist, the determinant of the coefficient matrix must be equal to zero, hence

$$0 = \alpha(\theta-k)^2 - \alpha\gamma^2 \tag{4.56}$$

For α not equal to zero, equation (4.56) implies that there are two roots to the characteristic equation:

$$k = \theta + \gamma \tag{4.57}$$

$$k = \theta - \gamma \tag{4.58}$$

Note that the reaction terms relating countries A and B are not important in this case. There is one more characteristic root that can be found from a slightly different statement of the problem. The root comes from the system of equations that can be expressed as

$$\begin{bmatrix} \alpha & \beta & -k \\ \gamma & 0 & (\ -k) \\ -k & \alpha & \beta \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \tag{4.59}$$

The characteristic roots must solve

$$0 = \beta k^2 + (\alpha^2 - \alpha\gamma - \beta\theta)k - (\alpha^2\theta + \gamma\beta^2) \tag{4.60}$$

Since this system is not independent of the first, and as the numerical results indicate, one of these roots is always equal to one of the roots in the first case, and the other is distinct. Hence, there are three roots to the complete system. In the limits, only the first two are necessary to demonstrate stability. That is, since γ and θ are related such that they are either equal or opposite, one of the roots from equations (4.57) and (4.58) must be zero. That is, the system is partially degenerate. Secondly, unless the absolute value of p is less than one-half, the system will be degenerate. As will be seen within the analysis of the individual cases, the lowest limiting value is one-half. Hence, the system is unstable in the limit. It is also fairly clear that the system is unstable for each of the individual cases, as will be shown numerically.

Case 1

The first case is based on setting γ equal to the sum of α and θ. The value for β is then equal to twice that of θ. These values are all greater than zero. For the most part, the solutions to the system evaluated as the number of importers and exporters increases are more important than any individual solutions. However, it is easier to coordinate the two solutions by considering a base system. Most of the numerical results were generated with the following data:

$$a_1 = 10 \tag{4.61}$$

$$a_2 = -5 \tag{4.62}$$

$$b_1 = 2 \tag{4.63}$$

$$\delta = 0.5 \tag{4.64}$$

These values were chosen because the round off error generated in solving the equations was fairly small. Other numbers were tested (particularly for δ and b_1) and they generated similar results.

Substituting these values into equation (4.42) implies that as n approaches infinity,

$$\gamma = \sqrt{2}\,/2 \tag{4.65}$$

As mentioned above, the tariff rates are indeterminate, but the numerical results (summarized in table 4.1) indicate that the tariffs eventually approach zero.

Table 4.1. Coefficients for Case 1

number of sellers		number of buyers			
		2	3	4	5
1	α	0.25	0.22	0.19	0.16
	θ	0.88	0.80	0.76	0.74
	β	1.76	1.60	1.53	1.48
	γ	1.13	1.02	0.95	0.91
	Y	-0.78	-0.92	-0.98	-1.01
	S	0.56	0.67	0.74	0.79
	T*	-1.56	-1.01	-0.82	-0.70
2	α	0.16	0.18	0.17	0.15
	θ	0.96	0.92	0.87	0.83
	β	1.92	1.83	1.74	1.67
	γ	1.12	1.10	1.03	0.98
	Y	-0.73	-0.81	-0.87	-0.91
	S	0.60	0.65	0.70	0.74
	T*	-3.80	-2.28	-1.67	-1.36
3	α	0.10	0.15	0.14	0.13
	θ	0.95	0.97	0.93	0.90
	β	1.91	1.93	1.87	1.80
	γ	1.05	1.11	1.08	1.03
	Y	-0.75	-0.77	-0.80	-0.84
	S	0.67	0.66	0.68	0.71
	T*	-5.95	-3.75	-2.65	-2.07
4	α	0.07	0.10	0.12	0.12
	θ	0.93	0.97	0.97	0.95
	β	1.85	1.95	1.94	1.90
	γ	0.99	1.08	1.09	1.06
	Y	-0.78	-0.76	-0.77	-0.80
	S	0.72	0.68	0.68	0.70
	T*	-7.79	-5.26	-3.73	-2.87

$\gamma = (\alpha+\theta) > 0$ $\qquad$ $\delta = 0.5$

The first four tables consist of the values of the coefficients of the reaction functions and the optimal tariffs, generated for various numbers of buyers and sellers in the market. On the other side, as the number of sellers (m) increases, γ decreases to one-half. Hence, the optimal tariff approaches minus infinity. That is, if the number of exporters is increased, without increasing the number of importers, each country will offer a larger and larger subsidy. Finally, increasing both the number of exporters and importers, γ approaches one, and the optimal tariff becomes zero.

The implication for stability is that of the three roots, one approaches zero, one is always greater than one (in absolute value), and the absolute value of the third is slightly less than one. Hence, the solution is a saddle point.

Case 2

The second case is similar to the first, in that γ equals the sum of α and θ; but now they are all less than zero. As the number of importers is increased,

$$\gamma = \sqrt{2}/2 \tag{4.66}$$

which is the negative of the first case. Again, the tariff level cannot be determined analytically. The numerical results (table 4.2) indicate that the tariff level is not highly responsive to changes in either n or m. That is, although the analytical results indicate that the tariff can approach a large subsidy as m is increased, it requires a much larger m in this case than in the first one.

The analysis of stability is very similar to that in the first case. The primary difference is in the sign of the roots. However, since the sign of a characteristic root is not important--except to indicate oscillation--the second solution is also a saddle point.

Case 3

The third case comes from setting γ equal to the negative of the sum of α and θ. In this case, γ is negative, and the sum of α and θ is positive. The numerical results are indicated in table 4.3. Concerning the reaction coefficients, the only difference in this case comes from the signs. Most of the limit analysis is the same. However, the level of the tariff can be much different. For most values of the discount factor, the tariff will still be negative. However, as indicated in equation (4.47); used in calculating ε as m becomes large; it is possible for ε

Table 4.2. Coefficients for Case 2

number of sellers		number of buyers			
		2	3	4	5
1	α	-0.25	-0.22	-0.19	-0.16
	θ	-0.88	-0.80	-0.76	-0.74
	β	-1.76	-1.60	-1.53	-1.48
	γ	-1.13	-1.02	-0.95	-0.91
	Y	-0.78	-0.92	-0.98	-1.01
	S	0.56	0.67	0.74	0.79
	T*	-0.86	-1.61	-0.98	-0.90
2	α	-0.17	-0.18	-0.17	-0.15
	θ	-0.96	-0.92	-0.87	-0.83
	β	-1.92	-1.83	-1.73	-1.67
	γ	-1.12	-1.10	-1.03	-0.98
	Y	-0.73	-0.81	-0.87	-0.91
	S	0.60	0.65	0.70	0.74
	T*	-1.20	-1.31	-1.34	-1.31
3	α	-0.10	-0.14	-0.14	-0.13
	θ	-0.95	-0.97	-0.94	-0.90
	β	-1.91	-1.93	-1.87	-1.80
	γ	-1.05	-1.11	-1.08	-1.63
	Y	-0.75	-0.77	-0.80	-0.84
	S	0.67	0.66	0.68	0.71
	T*	-1.52	-1.50	-1.52	-1.51
4	α	-0.07	-0.10	-0.12	-0.12
	θ	-0.93	-0.97	-0.97	-0.95
	β	-1.85	-1.95	-1.94	-1.89
	γ	-0.99	-1.08	-1.09	-1.06
	Y	-0.78	-0.76	-0.77	-0.80
	S	0.72	0.68	0.68	0.69
	T*	-1.80	-1.69	-1.65	-1.64

$\gamma = (\alpha+\theta) < 0$ $\qquad \delta = 0.5$

Table 4.3. Coefficients for Case 3

number of sellers		number of buyers			
		2	3	4	5
1	α	0.14	0.14	0.13	0.12
	θ	0.65	0.64	0.64	0.65
	β	-1.29	-1.28	-1.28	-1.29
	γ	-0.77	-0.77	-0.77	-0.77
	Y	-1.16	-1.19	-1.19	-1.18
	S	-0.94	-0.94	-0.95	-0.95
	T*	-0.62	-0.94	-1.07	-1.11
2	α	0.07	0.09	0.09	0.09
	θ	0.67	0.66	0.66	0.66
	β	-1.34	-1.33	-1.32	-1.32
	γ	-0.74	-0.75	-0.75	-0.75
	Y	-1.18	-1.19	-1.19	-1.18
	S	-1.06	-1.03	-1.02	-1.02
	T*	-0.96	-1.25	-1.40	-1.48
3	α	0.05	0.06	0.07	0.07
	θ	0.68	0.68	0.67	0.67
	β	-1.37	-.135	-1.34	-1.34
	γ	-0.73	-0.74	-0.74	-0.74
	Y	-1.17	-1.19	-1.19	-1.18
	S	-1.09	-1.08	-1.07	-1.06
	T*	-1.15	-1.40	-1.56	-1.65
4	α	0.03	0.05	0.05	0.06
	θ	0.70	0.68	0.68	0.68
	β	-1.38	-1.37	-1.36	-1.36
	γ	-0.72	-0.73	-0.73	-0.74
	Y	-1.16	-1.18	-1.18	-1.18
	S	-1.11	-1.09	-1.09	-1.08
	T*	-1.27	-1.49	-1.64	-1.74

$\gamma = -(\alpha+\theta) > 0 \qquad \delta = 0.5$

to be positive, which means that if the discount factor is sufficiently high, the optimal tariff will be positive. As m and n increase, it will eventually become negative, and then approach zero. This case is demonstrated in table 4.5, which was generated with a discount factor of 0.8.

Once again, this solution is a saddle point. Changing the signs does not affect stability. There is a root greater than one; one that approaches zero, and one that is between zero and negative one.

Case 4

Set four is generated in a manner similar to set three in that γ is equal to the negative of the sum of θ and α. However, the signs are reversed, and γ is greater than zero. Again, as n gets large, it appears that the optimal tariffs approach zero. However, as m increases, the optimal tariff must be a subsidy. Since γ is greater than zero, equation (4.47) must always be negative. Also, when both m and n get large, the optimal tariff becomes zero. These changes can be observed in table 4.4.

Like the other cases, this solution is also a saddle point. That is, one characteristic root

Table 4.4. Coefficients for Case 4

number of sellers		number of buyers			
		2	3	4	5
1	α	-0.12	-0.14	-0.13	-0.12
	θ	-0.65	-0.64	-0.64	-0.65
	β	1.29	1.28	1.28	1.29
	γ	0.77	0.77	0.77	0.77
	Y	-1.16	-1.19	-1.19	-1.18
	S	-0.94	-0.94	-0.95	-0.95
	T*	-1.94	-3.04	-1.37	-1.19
2	α	-0.07	-0.09	-0.09	-0.09
	θ	-0.67	-0.66	-0.6	-0.66
	β	1.34	1.33	1.32	1.32
	γ	0.74	0.75	0.75	0.75
	Y	-1.18	-1.19	-1.19	-1.18
	S	-1.06	-103	-1.03	-1.02
	T*	-3.51	-2.87	-2.44	-2.13
3	α	-0.05	-0.06	-0.07	-0.07
	θ	-0.68	-0.68	-0.67	-.067
	β	1.37	1.35	1.35	1.34
	γ	0.73	0.74	0.74	0.74
	Y	-1.17	-1.19	-1.19	-1.18
	S	-1.09	-1.08	-1.07	-1.06
	T*	-4.82	-3.95	-3.36	-2.94
4	α	-0.03	-0.05	-0.05	-0.06
	θ	-0.69	-0.68	-0.68	-0.68
	β	1.38	1.37	1.38	1.36
	γ	0.72	0.73	0.73	0.74
	Y	-1.16	-1.18	-1.18	-1.18
	S	-1.11	-1.09	-1.09	-1.08
	T*	-5.90	-4.88	-4.18	-3.67

$\gamma = -(\alpha + \theta) < 0$ $\delta = 0.5$

Table 4.5. Comparison of Tariffs

number of sellers		number of buyers		
		2	4	9
1	one	-13.30	-0.14	-0.01
	two	-0.61	-0.83	-0.56
	three	-2.47	-2.38	-1.08
	four	-2.06	-1.43	-0.82
	Cournot	-1.56	-1.25	-0.71
4	one	-10.50	-3.30	-0.65
	two	-1.42	-1.22	-1.21
	three	1.05	1.09	-2.69
	four	-6.89	-4.61	-2.69
	Cournot	-3.68	-3.13	-2.13
9	one	-22.70	-11.90	-2.90
	two	-2.19	-1.79	-1.41
	three	0.77	0.75	0.81
	four	-1.17	-8.37	-5.09
	Cournot	-5.10	-4.53	-3.47

Tariffs calculated for $\delta = 0.8$

is less than negative one; one approaches zero, and the other is between zero and one. Because of the large negative root, the system cannot return to its original position if it is perturbed.

Comparison of Cases

The tariffs for the various cases are presented in tables 4.5 and 4.6. These tables consist of the optimal tariffs for each of the five cases with various numbers of buyers and sellers in the market. In comparing the cases, it is important to consider a few general comments first. To begin with, the five solutions (four cases plus Cournot) are all very similar. As both m and n get large, the optimal tariffs approach zero. In fact, in most cases, for a fixed number of exporters, increasing the number of importers causes an optimal subsidy to decrease monotonically to zero. Note that it is possible for the subsidies to start at a lower level, increase, and then decrease to zero. However, in the numerical results considered, the decrease usually begins before there are ten importers in the market. Second, as the number of exporters is increased, the level of the optimal subsidy generally increases. Finally, with the exception of set three, the solutions

Table 4.6. Comparison of Tariffs

number of sellers		number of buyers		
		2	4	9
1	one	-1.56	-0.81	-0.47
	two	-0.86	-0.98	-0.62
	three	-0.63	-1.07	-1.01
	four	-1.94	-1.37	-0.77
	Cournot	-1.56	-1.25	-0.71
4	one	-7.79	-3.73	-1.60
	two	-1.80	-1.65	-1.49
	three	-1.27	-1.64	-1.89
	four	-5.90	-4.18	-2.51
	Cournot	-3.68	-3.13	-2.13
9	one	-13.75	-8.90	-3.72
	two	-2.61	-2.26	-1.90
	three	-1.51	-1.76	-2.09
	four	-9.53	-7.18	-4.60
	Cournot	-5.10	-4.53	-3.47

Tariffs calculated for $\delta = 0.5$

are not highly responsive to changes in the discount factor. Some of the tariffs change, but the same general pattern remains. Essentially, this result seems to imply that the most important variable in the system is the number of exporters and importers.

Although the five cases generate similar solutions, they come from different assumptions about how the other countries involved will react. Cournot assumes that if a tariff is changed, there will be no reaction by either type of country. Note that as the number of both types of countries increases, the Cournot case becomes a more "realistic" assumption, and all of the solutions tend toward the free trade results.

For the other cases, examine a market where the number of importers and exporters is small enough so that a change in a tariff could generate a measurable reaction. These other cases are generated by considering how an importing country will respond to a change in its own last period tariff (indicated by the sign of γ), the tariff imposed by the other importing countries in the last period (α), and the last period tariff imposed by the exporting countries (θ or β). The first case is the most straight-forward, in that if any country increased its tariff in the last period, country

A will be inclined to increase its tariff in this period. Hence, this case could be called "full retaliation." That is, all countries know that if one country increases its tariff, the others will retaliate by increasing their own tariffs, hence, decreasing any possible gain. In general terms, this solution seems to generate the largest subsidy. However, as the number of buyers increases, this subsidy appears to drop to zero faster than in the other cases.

In the second case, country A will decrease its tariff if the exporting countries raise their tariffs. Presumably this action is an attempt to generate lower prices for the consumers to maintain their welfare. More importantly, if the other importing countries increase their tariff, country A will decrease its tariff, thus gaining a larger share of the consuming market. Although this case tends to generate the lowest subsidy, it is also the most consistent. That is, changing the number of buyers or sellers in the market has little effect on the optimal tariff--perhaps because the existing countries are already behaving in a "competitive" fashion. That is, rather than challenging the other countries, and threatening to increase tariffs, country A moves in the opposite direction to "protect" the current level of trade and welfare.

In the third case, the tariffs of the importing country A move in the opposite direction of those imposed by the exporting countries. Which means that country A is attempting to maintain its position vis-a-vis its suppliers. However, if the other importing countries attempt to increase their tariffs, country A threatens to retaliate with a higher tariff. As noted above, this case can actually lead to a positive tariff if the discount factor (δ) is high enough. Apparently, if society places a high value on future welfare; and there is a small number of exporters and importers; the countries can gain more from the tariff revenue than from the respective surplus approximation to welfare. Observe that as the number of either buyers or sellers increases (above five or six in this example), this power over price rapidly diminishes, and this case will generate the largest subsidy.

The final case is similar to the second case, in that country A will attempt to protect its current position by moving in the opposite direction of tariffs imposed by any of the other countries. However, if country A increased its tariff in the last period, they will have a type of "momentum" and continue to increase their tariff. Also, the other importing countries will (in total) change tariffs in the same direction as the exporting countries. Hence, the importing countries as a group will appear to be threatening to increase their tariffs if the exporting countries increase their tariffs. Therefore, this case will behave in a manner similar to the first case; except there is a damping effect.

Conclusion

It is important to note that in almost all of the cases, the optimal tariff is negative. Although the particular behavior of the countries towards each other affects the level of the subsidy, the best action is still a subsidy. The key to understanding this point is that it is not profitable for a country to unilaterally grant a subsidy. It is only through perfect foresight--when each country knows the other will "retaliate"--that all of the countries gain by granting a subsidy.

Perhaps the most important conclusion that can be derived from the results is that the solutions behave as they might be expected to behave. That is, as the market gains exporters and importers, the system approaches free trade. In fact, the conclusion is even stronger. As the comparison tables clearly show, the most important variables in the system are the number of participants. Although there are five types of behavior that represent perfect foresight solutions, they all generate very similar results. This conclusion seems to suggest that although the individual behavior of a country will affect the welfare of the "world;" any effect is minimal unless there are very few countries involved.

CHAPTER 5. RETALIATION WITH UNCERTAINTY

The primary focus of the analysis to this point has been to model the behavior of countries that possess perfect foresight about how their "opponents" will respond to changes in tariffs. A question that seems to follow naturally from that analysis is to ask whether or not it is possible to model a situation where the countries possess some degree of foresight, but there may be an element of error in this foresight. Although the question flows easily from the last topic, the answer is much more complicated.

When dealing with uncertainty, the overriding question that must be answered is: Why does the uncertainty exist? To understand the importance of this question, consider a situation in which country one does not know (with certainty) how country two will react. That is, country one starts with a belief about how two will react, but this belief is not necessarily correct. Given this belief, country one will impose its best tariff. Based on country one's tariff, country two will then impose some tariff (possibly no change). The point is that this action by country two provides new information that country one can use to update its belief function. If this retaliation process continues, country one should eventually be able to learn two's "true" reaction function. A process similar to the one described here is precisely what was used to generate the perfect foresight solutions in Chapters 3 and 4. In other words, even if country one is initially uncertain about two's response, the retaliation process is a learning process that will eventually allow one to determine two's response to any tariff. Hence, the question remains, why would uncertainty exist?

There are essentially three possible reasons why a country may be uncertain about how another will react to tariff changes. First, both countries may be at the start of the retaliation process, and hence, would not yet know how the other will respond. Second, a country may be intentionally introducing an element of randomness into the retaliation process. That is, once the optimal tariff has been decided upon, a random component is added in, causing a slightly different tariff to appear each time. Finally, there may be exogenous factors causing uncertainty. For example, a country's agricultural exports may depend heavily on the weather, hence the export supply curve would contain a random element. This component could affect either the intercept term, or the slope of the supply curve. Consider the three cases individually.

Initial Uncertainty

As can be seen from the analysis in Chapter 3, a country may not initially know how another country will respond to changes in tariffs. Eventually, a retaliation process will generate a belief function that is "correct;" but at the start, a country does not really "know" how the other will respond.

An argument of this type is somewhat sophistic, because the retaliation process in Chapter 3 never really occurs. That is, to generate the perfect foresight results, it is only necessary that each country be rational--in the sense that they maximize some measure of welfare. As a result, there are only five possible equilibrium points. At the beginning of the process, the only possible uncertainty that can exist is which type of reaction the other country will pursue: no reaction, full retaliation, or opposite retaliation. Even in the "real world," it should be fairly straight-forward to decide which process the "opponent" will utilize. Therefore, there can be no initial uncertainty if each country is rational and maximizes some measure of welfare, and also knows that the other country is rational.

Intentionally Introduced Uncertainty

Uncertainty that may be introduced by an individual country is perhaps the most difficult to comprehend. The case is considered here primarily for the sake of completeness. The manner in which it could be introduced can be explained as follows. Country two begins with some belief about how country one will respond. Then they maximize some welfare function and come up with an optimal tariff to impose. Now, they randomly generate an error term (presumably from a fixed distribution) to be added to this tariff. In other words, their reaction function would consist of four components. A constant term, a term containing their last period tariff, a term containing country one's last tariff, and an error term. If the other country (country one) is going to possess foresight, they would have to know the first three terms with certainty, and they would know the distribution from which the error term is drawn. The effect of this error term is that country two's retaliation would always vary by some amount. From the perspective of country one, there are two important consequences. First, the error term would make it more difficult to "learn" the reaction function of country two. Secondly, there will always be an element of uncertainty about the actual tariff level.

The ultimate question is what does country two gain by introducing this uncertainty? At first glance, it seems that making it more difficult for country one to learn the process may provide a means to acquire additional welfare. However, note that because there is no actual retaliation process, it does not matter how many periods are required to determine the "actual" response functions. That is, given the rationality assumptions, the retaliation process occurs instantaneously. From a slightly different perspective, the system utilizes an infinite time horizon, so any "delay" would be irrelevant.

The question as to whether or not country two can gain by introducing an error term depends on the degree to which the uncertainty about the final tariff can influence country one's decision. That is, if the error term has a mean of zero, intuitively, the best decision that country one can make should be based on the reaction function without the error term. The only problem is that the final decision may also be based on the variance of the error term. This dependence on the variance may prove to be an interesting topic for further research, but is outside the scope of this paper for several reasons. First, the nature of the retaliation has now changed from perfect foresight to a situation where a country is trying to deceive another country. The essential question then shifts from "What is the optimal level of tariff to impose?" to "What is the optimal form of deceit to use?". Secondly, the dynamic programming methodology may not be the best way to model the uncertainty. For example, it may be more accurate to use a Bayesian process in which the distribution function itself is estimated.

Exogenous Uncertainty

Within the two country model, it is interesting to examine what would happen if the production, or consumption functions were subject to some degree of uncertainty. That is, for some purely exogenous reason (outside the control of either country), the supply or demand curves may change in any given time period. For example, an agricultural sector may be subject to fluctuations in the weather, hence, exports may be affected by a random component. Alternatively, changes in income may influence the level of imports.

In the model presented in chapter 3, there are two possible ways to include this uncertainty. First, the intercept terms for the excess supply and demand curves may have a random shift term, or, the slope may have a random shift parameter. The only problem with introducing exogenous uncertainty into this model is that the results are negligible. That is, because of the linearity of the model, the variance term drops out, and since the mean is zero, the conclusions are unchanged. However, as an example, it shows how this uncertainty can be modelled.

Consider uncertainty about the intercept terms. Let e_1 and e_2, be two independent random variables with means of zero, and variances denoted s_1^2 and s_2^2. Then, the excess supply and demand curves can be written

$$Qs = (a_1 + e_1) + b_1 P \tag{5.1}$$

$$Qd = (a_2 + e_2) - b_1(P+T_1+T_2) \tag{5.2}$$

Based on the algebra presented in Chapter 3 (delineated in Appendix A), it is fairly easy to show that for country one (the importing country), the welfare approximation can be written

$$U_1 = a + bT_1 + cT_2 + gT_1T_2 + eT_1^2 + fT_2^2 \qquad (5.3)$$

where

$$a = (a_1+e_1 + a_2+e_2)^2/8b_1 \qquad (5.4)$$

$$b = (a_1+e_1 + a_2+e_2)/4 \qquad (5.5)$$

$$c = -(a_1+e_1 + a_2+e_2)/4 \qquad (5.6)$$

$$g = -b_1/4 \qquad (5.7)$$

$$e = -3b_1/8 \qquad (5.8)$$

$$f = b_1/8 \qquad (5.9)$$

Observe that the last three terms do not depend on the random elements. The next step is to take the expected value of this welfare measure, and then to maximize it under the model presented in Chapter 3. Since the means of the random terms are zero, taking the expected value eliminates the random element from all but the (a) coefficient which becomes

$$a = [(a_1 + a_2)^2 + (s_1^2 + s_2^2)]/8b_1 \qquad (5.10)$$

All of the other coefficients have exactly the same form as they did in the model without uncertainty.

Note that each country still possesses as much information as the other country. That is, there is still perfect foresight about the other country's reaction function; up to any degree of uncertainty. Hence, if one country is uncertain about a reaction, it is only because of an exogenous reason, and both countries are equally uncertain. Note that the system was solved in Appendix A in general terms for any coefficients (a-g). Recall that there were a total of five solutions in three sets. First, the values satisfying the perfect foresight reaction functions, and the optimal tariff, for the Cournot case are

$$\alpha = \beta = 0 \tag{5.11}$$

$$\mu = -b/(g + 2e) \tag{5.12}$$

$$T^* = \mu \tag{5.13}$$

For the first two cases, the values are

$$\alpha = \beta = \pm \sqrt{1/2\delta} \tag{5.14}$$

$$\mu = (\alpha\delta c - b - \alpha\delta c)/\alpha\delta b_1 \tag{5.15}$$

$$T^* = \mu/(1 - 2\alpha) \tag{5.16}$$

In the last two cases, the values are

$$\alpha = -\beta = \pm \sqrt{1/4\delta} \tag{5.17}$$

$$\mu = -b/(2g(1+2\alpha\delta) - (f-e)) \tag{5.18}$$

$$T^* = \mu \tag{5.19}$$

Note that the constant term (a) from the welfare equation does not appear in any of the results. Hence, the uncertainty has no role in deciding the optimal tariff. This conclusion can also be verified from standard micro-economic theory. If a welfare function is modified by a constant term, the optimal decision will not be affected, since all of the decisions are made on the basis of the slope (marginal) of the function. Note that the expected level of welfare will vary, depending on the variance of the random terms.

The other way that uncertainty can be introduced is to consider that the slope may have a random component. Treating this element as a multiplicative element, the slope could be expressed as

$$e_1 b_1 \tag{5.20}$$

Where e_1 is a term randomly distributed about one, with a variance denoted s_1^2. Recall that the constant (a) term in the welfare function can be ignored, so consider the other terms that contain b_1. From equations (5.3) through (5.9), it is clear that it only enters into coefficients g, e, and f. Further, since the excess supply and demand functions are linear, the b_1 term only enters into these welfare coefficients in a linear fashion. Hence, as soon as expected welfare is considered, the random component e_1 will drop out. Therefore, the optimal tariff will not be affected by this random component either.

Conclusion

The only important conclusion from this analysis is that in the linear model considered, exogenous uncertainty will not affect the level of the optimal tariffs. This conclusion may seem fairly trivial, but it could have some ramifications. In a "realistic" situation, various exogenous uncertainties exist. From the above analysis, it is only necessary to have an unbiased estimate of the parameters. Then, the optimal tariff will be the same regardless of the degree of uncertainty--measured by the variance. One question that has not been answered is how exogenous uncertainty would affect the solutions in a model that does not rely on linear excess supply and demand curves. This paper does not seem the appropriate place to attempt an answer, since all of the previous analyses rest on this assumption. Further, within some neighborhood of an equilibrium solution, the linear approximation should normally be fairly accurate.

CHAPTER 6. CONCLUSION

The historical approaches to optimal tariffs have relied on assuming that at least one country has a naive belief about how the other country will react to changes in tariffs. The objective of this study has been to extend that analysis by examining solutions in which both countries know how the other will react. It turns out that as long as there are no adjustment costs, five solutions are consistent with perfect foresight. The first case is the well-known Cournot solution, where each country believes that the other will not change its tariffs in response to a change by the other country. In this case, there is a pair of tariffs where this assumption holds, and neither country does alter its tariff schedule. In many ways, the Cournot tariff is the focal point for the other solutions. When just considering two countries, there are two sets tariffs that are higher than the Cournot tariff, and two that are lower. Each of these four sets is generated by a different assumption about how the other country will react. There are two primary classifications that generate these solutions. In the first, each country believes that if they impose a tariff, the other country will retaliate and impose a similar tariff, thus directly attacking the original tariff. In the second case, if one country were to impose a higher tariff, the other country would "retaliate" by imposing a tariff that is precisely the opposite of the original tariff, thus negating the effect of that tariff.

The importance of the study comes from its ability to give all countries involved an "intelligent" reaction. That is, in some sense, it is more realistic in that if countries did happen to get into a tariff war, it seems reasonable that each country would eventually learn the other's retaliation process. In other words, over time it does not seem reasonable that a country can remain ignorant about the other's reactions. The main point from the study is that perfect foresight solutions actually exist. Previous literature has essentially treated the problem by claiming that there is no determinate solution.

The second interesting point generated from the study is that perfect foresight solutions for more than two countries behave as they would be "expected" to behave. As the number of exporters and importers increases, the solution approaches free trade. Which means that as the number of participants in the market increases, the asset of perfect foresight becomes less valuable. When the number of countries is relatively large, they all know how the others are going to respond, but they cannot take advantage of this knowledge because they have no market power.

As a final point, it should be noted that the study is far from being a complete analysis of the perfect foresight problem. There are many individual situations to which the analysis could be applied. Such as optimal tariffs with preferential treatment to certain nations. It may also be useful to consider non-linear supply and demand curves--especially to examine the effects of exogenous uncertainty on the optimal tariffs. However, it would still be necessary to retain the symmetry between the exporting and importing countries in order to arrive at determinate solutions. Another possible area for expansion is to relax the symmetry assumption. Of course, as soon as this assumption is removed, an infinite number of solutions appear. However, it is possible that other constraining equations could be imposed. That is, a particular country may be operating under institutional constraints that would help impose a specific solution. Finally, it may be possible to expand the analysis even further by considering a country that has control over both price and output; possibly through a state trading agency; so that an optimal reaction function could be generated in both spaces.

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APPENDIX A. DERIVATION OF OPTIMAL TARIFFS

This appendix presents the details of the algebraic derivation of the optimal tariff for countries one and two in a dynamic programming framework. The limiting assumptions are that the excess supply and demand curves are linear, and the absolute value of their slopes are equal. The first step is to show that as a result of these assumptions, the optimal tariffs will be symmetric. Using this result and perfect foresight, the actual levels of the optimal tariffs can be found.

In algebraic terms, the supply curve is given by:

$$Qs = a_1+b_1P \tag{A1}$$

For country one imposing an export tariff (T_1) and country two imposing an import tariff (T_2), the demand curve facing the consumers in country two is given by:

$$Qd = a_2-b_1(P+T_1+T_2) \tag{A2}$$

These two equations can be solved for an equilibrium price to the sellers in country one (Pe) and equilibrium quantity (Qe) expressed in terms of T_1 and T_2.

$$Pe = [a_2-a_1-b_1(T_1+T_2)]/2b_1 \tag{A3}$$

$$Qe = [a_1+a_2-b_1(T_1+T_2)]/2 \tag{A4}$$

Further, since they will be needed in the calculation of the net surplus, it can be shown that

$$Pe+a_1/b_1 = Qe/b_1 \tag{A5}$$

and

$$a_2/b_1-(Pe+T_1+T_2) = Qe/b_1 \tag{A6}$$

These results follow directly from the definitions of the supply and demand curves--just divide each side by b_1.

As in figure 2.1, the value of a tariff to country one can be approximated as the sum of the tariff revenue, and the net producers' surplus. In notational form,

$$U_1 = Qe/2\ (Pe+a_1/b_1) + T_1Qe \tag{A7}$$

$$= 1/(2b_1)\ Qe^2 + T_1Qe \tag{A8}$$

Substituting in the values for Qe from (A4) yields

$$\begin{aligned} U_1 = {} & (1/8b_1)(a_1+a_2)^2 + (1/4)(a_1+a_2)T_1 \\ & -(1/4)(a_1+a_2)T_2 - (b_1/4)T_1T_2 \\ & -(3/8)b_1T_1^2 + (1/8)b_1T_2^2 \end{aligned} \tag{A9}$$

To make notation easier, the variables a-g will be defined as the above coefficients such that

$$\begin{aligned} U_1 = {} & a + bT_1 + cT_2 + gT_1T_2 \\ & + eT_1^2 + fT_2^2 \end{aligned} \tag{A10}$$

In a similar manner, it can be shown that for country two,

$$U_2 = (1/2)Qe(a_2/b_1-(T_1+T_2+Pe)) + T_2Qe \qquad \text{(A11)}$$

Which, using (A6) reduces to

$$U_2 = (1/2b_1)Qe^2 + T_2Qe \qquad \text{(A12)}$$

Equation (A12) is clearly symmetric to equation (A8) with respect to T_1 and T_2. Therefore, if country two follows the same dynamic programming optimization process as country one, the optimal tariffs that are generated must be symmetric with respect to T_1 and T_2.

From the perspective of country one, the dynamic programming problem can be expressed in the form of two equations. All actions shall be delineated by some time period denoted as (t). For country one, the state variable will be country two's tariff $T_2(t)$. The control variable available to country one is its tariff $T_1(t)$. The essence of the model is that country one cannot influence the current state variable. That is, it cannot change the current $T_2(t)$. However, country two believes that it can influence future levels of T_2. The system equation which defines the movement of the state variable is

$$T_2(t+1) = \mu + \alpha T_1(t) + \beta T_2(t) \qquad \text{(A13)}$$

The coefficients μ, α, and β are assumed to be known by country one, but are essentially variables at this point.

The objective of country one is to maximize its surplus (U_1) subject to equation (A13). Hence, the performance criterion can be written as

$$K[T_2(t)] = (a+bT_1^*+cT_2+gT_1^*T_2+eT_1^{*2}+fT_2^2) + \delta K[T_2(t+1)] \tag{A14}$$

Basically, $K[T_2(t)]$ is the present value of the current surplus, and future returns discounted by the variable δ.

These two equations define the dynamic programming problem. The procedure is to first find an expression for the function K, and then maximize it with respect to $T_1(t)$ and solve for the optimal $T_1(t)^*$ in terms of $T_2(t)$.

In general terms, the problem can be solved from the next four equations. Differentiating equation (A14) with respect to $T_1(t)$, and noting that $T_2(t)$ is fixed with respect to $T_1(t)$ yields

$$b+gT_2(t)+2eT_1(t)+\alpha\delta K'(t+1) = 0 \tag{A15}$$

If the function K were known, this equation could be solved for $T_1(t)^*$. Totally differentiating equation (A15) yields

$$0 = (g)dT_2(t) + (2e)dT_1(t)$$
$$+ \alpha\delta K''(\alpha dT_1(t) + \beta dT_2(t)) \qquad (A16)$$

In order to find the function K, it is necessary to differentiate (A14) with respect to $T_2(t)$, then, applying the envelope theorem yields

$$K'(t) = c+gT_1*+2fT_2+\beta\delta K'(t+1) \qquad (A17)$$

Taking the second derivative generates

$$K''(t) = 2f+\beta^2\delta K''(t+1)$$
$$+ (\alpha\beta\delta K''+g)dT_1*/dT_2 \qquad (A18)$$

The next basic step is to state that the function K can be expressed as quadratic in T_2. This may seem somewhat arbitrary, but it will be shown to be accurate below. The reason it works is because the surplus function (U_1) is quadratic in T_1 and T_2. In notational form, for B_0, and B_1 as yet unknown,

$$K(t) = B_0T_2(t) + B_1T_2(t)^2 \qquad (A19)$$

Combining (A14) and (A19) yields

$$(b+\alpha\delta B_0+2\alpha\delta\mu B_1) + (2e+2\alpha^2\delta B_1)T_1(t)$$
$$+ (g+2\alpha\delta\beta B_1)T_2(t) = 0 \qquad (A20)$$

To simplify the notation, define H,Y,S so that

$$H + YT_1(t) + ST_2(t) = 0 \qquad (A21)$$

As a result, accepting B_1 as known, (A21) can be solved for

$$T_1(t)^* = -H/Y - (S/Y)T_2(t) \tag{A22}$$

which when differentiated solves for

$$dT_1^*/dT_2 = -(S/Y) \tag{A23}$$

By using the state equation (A13), (A22) transforms to

$$T_1(t)^* = (-H-S\mu)/Y - \alpha(S/Y)T_1(t-1) - \beta(S/Y)T_2(t-1) \tag{A24}$$

Equation (A24) defines the optimal T_1 in terms of the last period's T_1 and T_2. In order to find B_1, it is necessary to combine equations (A19) and (A18) to get

$$2B_1 = 2f+\delta\beta^2(2B_1) + (g+2\alpha\delta\beta B_1)dT_1^*/dT_2 \tag{A25}$$

Substituting in for dT_1^*/dT_2 from (A23) yields an equation which can be solved for B_1:

$$2B_1(1-\delta\beta^2) = 2f - (S^2/Y^2)Y \tag{A26}$$

Note that B_1 is a function solely of δ, α, β, and not of $T_2(t)$. That is, the original assumption that K was quadratic in $T_2(t)$ has been upheld.

The next basic step is to solve for B_0, the linear term in the the system equation. Equations (A17) and (A19) imply

$$B_0+2B_1T_2(t) = c+gT_1(t)+2fT_2(t) + \delta\beta(B_0+2B_1T_2(t+1)) \tag{A27}$$

Using the defined variable (S), equation (A27) reduces to

$$(-S)T_1(t)^* = (c+2\delta\beta\mu B_1-(1-\delta\beta)B_0) + 2(f-(1-\delta\beta^2)B_1)T_2(t) \quad \text{(A28)}$$

Since there can be only one functional form for $T_1(t)^*$, the constant term from equation (A22) must be equal to the constant term from equation (A28), or:

$$H/Y = (c+2\delta\beta\alpha B_1-(1-\delta\beta)B_0)/S \quad \text{(A29)}$$

Noting that (H) has a B_0 term in it, equation (A29) can be solved for

$$B_0 = \frac{(c+2\delta\beta\mu B_1)-(b+2\alpha\delta\mu B_1)(S/Y)}{(\alpha\delta(S/Y) + (1-\delta\beta))} \quad \text{(A30)}$$

Notice that the function K has been determined. That is, B_0 and B_1 have been found as functions of $\delta, \alpha, \beta, \mu$. Also, $T_1(t)^*$ can be found as a function of $\delta, \alpha, \beta, \mu$. The problem now is to find α, β, μ. Imposing perfect foresight, μ, α, β must be known with certainty. By symmetry of the surplus functions for each country, equation (A24) must be symmetric to equation (A13) with respect to $T_1(t-1)$ and $T_2(t-1)$, since (A13) could have been derived by a similar procedure for country two. Therefore,

$$\mu = -(H+S\mu)/Y \quad \text{(A31)}$$

$$\alpha = -(S/Y)\beta \quad \text{(A32)}$$

$$\beta = -(S/Y)\alpha \quad \text{(A33)}$$

Combining equations (A32) and (A33) implies first that

$$S^2/Y^2 = 1 \tag{A34}$$

and secondly, that

$$\alpha^2 = \beta^2 \tag{A35}$$

Observe that; not counting the defined variables S,Y,H; there are now five equations: (A25), (A30), (A31), (A34), (A35); and five basic unknown variables: B_0, B_1, α, β, μ. However, recall that there are actually six defined variables (a-g), as denoted by equations (A9) and (A10). Hence, all five variables can be reduced to functions of the original model parameters a_1, a_2, b_1 and the discount rate δ.

There are two main cases from equations (A34) and (A35) that are fruitful, and the Cournot case. First, reduce (A34) and (A35) to

$$Y = -S \tag{A36}$$

$$\alpha = \beta \tag{A37}$$

App.ying (A37) reduces (A25) to

$$B_1 = f - e \tag{A38}$$

Inserting the values for f and e, yields

$$B_1 = b_1/2 \tag{A39}$$

Equation (A30) similarly simplifies to

$$B_0 = (c+b+4\alpha\delta\mu B_1)/(1-2\alpha\delta) \tag{A40}$$

Variable α can be found from equation (A36) by putting in the notation for S and Y:

$$\alpha^2 = -(2e+g)/(4\delta(f-e))$$

Which can be rewritten as

$$\alpha^2 = 1/2\delta \tag{A41}$$

Note that (A41) generates two values for α (and hence β).

Equation (A31) can similarly be reduced to

$$\mu = -(b-\alpha\delta b+\alpha\delta c)/(2\alpha\delta B_1) \tag{A42}$$

Using (A42), equation (A40) can be reduced to

$$\begin{aligned} B_0 &= c - b \\ &= -(1/2)(a_1+a_2) \end{aligned} \tag{A43}$$

Putting the values for a-g into equation (A42), μ can be expressed as

$$\mu = (1-\alpha)(a_1+a_2)/2b_1 \tag{A44}$$

Once the values for B_0, B_1, α, β, μ have been found, the steady state values for the optimal tariffs T_1* and T_2* can be found. In general form, using symmetry, recall that the tariffs can be expressed as

$$T_1(t)^* = \mu + \beta T_1(t-1) + \alpha T_2(t-1)$$

$$T_2(t)^* = \mu + \alpha T_1(t-1) + \beta T_2(t-1)$$

For a steady state solution to exist, the tariff rate in one period must be equal to the tariff rate in the next period, hence

$$T_1^* = \mu(1-\beta+\alpha)/(1-2\beta+\beta^2-\alpha^2) \tag{A45}$$

$$T_2^* = \mu/(1-\beta) + \alpha/(1-\beta)\ T_1^* \qquad \text{(A46)}$$

Applying equation (A45) to the values generated in the first case,

$$T_1^* = \mu/(1-2\alpha) \qquad \text{(A47)}$$

$$T_2^* = \mu/(1-2\alpha) \qquad \text{(A48)}$$

Equation (A48) verifies the symmetry. Note that μ and α are known, and there are two values for each one.

In the second major case, equations (A34) and (A35) are reduced to

$$\alpha = -\beta \qquad \text{(A49)}$$

$$Y = S \qquad \text{(A50)}$$

B_1 reduces to the same form as in the first case--equation (A39). Equation (A50) can be solved for α, such that

$$\alpha^2 = 1/4\delta \qquad \text{(A51)}$$

Note that β is the negation of α in this case, but once again there are two values for α and β. However,

$$B_0 = (c-b-4\alpha\delta\mu B_1)/(1+2\alpha\delta) \qquad \text{(A52)}$$

and

$$\mu = -(b+\alpha\delta B_0)/(g+2\alpha\delta B_1-2\alpha^2\delta B_1) \qquad \text{(A53)}$$

Equations (A52) and (A53) can be solved for B_0 and μ, which can then be reduced to

$$B_0 = -(1/2)(a_1+a_2) \qquad \text{(A54)}$$

$$\mu = (a_1+a_2)/2b_1 \qquad \text{(A55)}$$

Notice that μ does not depend on the value of α or β. Using the steady state results from (A45) and (A46), it follows that

$$T_1{*} = \mu \tag{A56}$$

$$T_2{*} = \mu \tag{A57}$$

Since μ does not depend on α or β, the steady state tariffs are also functionally independent of them.

For the final case, observe that the Cournot assumption is also a solution to the system of equations. For this case, α and β are both zero, and the other equations simplify easily to

$$B_1 = b_1/6 \tag{A58}$$

$$B_0 = -(1/3)(a_1+a_2) \tag{A59}$$

$$\mu = (a_1+a_2)/4b_1 \tag{A60}$$

Finally, the steady state tariffs in this case reduce to

$$T_1{*} = \mu \tag{A61}$$

$$T_2{*} = \mu \tag{A62}$$

These results appear similar to the second case, but note that the constant term (μ) is defined differently, hence the actual tariff level is different.

APPENDIX B. DERIVATION OF MULTICOUNTRY TARIFFS

This appendix presents the derivation of the optimal tariffs for more than two countries. The excess supply and demand curves are linear, and the absolute values of their slopes are all equal. All exporters are assumed to have identical excess supply curves, and all importing countries have identical excess demand curves. These limiting assumptions imply that all the countries have equal economic power against any one other country, and any reaction by one type of country is symmetric to reactions by the other countries. Exporting countries maximize producer surplus and tariff revenue accruing from exported goods. Similarly, importing countries maximize consumer surplus and tariff revenue accruing from imported goods.

Consider n buyers and m sellers. Using the above assumptions, and using P to represent world price, total world supply can be expressed as

$$Qs = ma_2 + mb_1(P-Tc) \tag{B1}$$

where Tc is the tariff imposed by the exporting countries. Since they are identical, from the perspective of the importing countries, there will be only one tariff. From the perspective of one importing country, there will be (n-1) other importing countries; imposing tariff Tb; whose demand can be expressed by

$$Qd = (n-1)a_1 - b_1(P+Tb) \tag{B2}$$

Finally, the demand by the individual importer imposing tariff Ta can be written

$$Qd = a_1 - b_1(P+Ta) \tag{B3}$$

Combining the two demands yields

$$Qd = (n)a_1 - (n)b_1P - (n-1)b_1Tb - b_1Ta \tag{B4}$$

Equating (B1) and (B4) and solving for P yields

$$Pw = \frac{[na_1-b_1(-mTc+Ta+(n-1)Tb)]}{(m+n)b_1} \tag{B5}$$

Substituting the value for Pw into equation (B3) generates the equilibrium quantity imported by the one importing country:

$$Qa = m(a_1+a_2)/(m+n) + b_1/(m+n) \cdot [(1-m-n)Ta + (n-1)Tb - mTc] \tag{B6}$$

The welfare that this particular country gains from trade can be approximated by the increase in consumers' surplus plus the tariff revenue (which is assumed to be redistributed in some efficient manner), that is,

$$Ua = (1/2b_1)Qa^2 + TaQa \tag{B7}$$

Using the equilibrium value for Qa from equation (B6) yields

$$\begin{aligned}
Ua = {} & [m/(m+n)]^2 \\
& + [m(m+n)-2mb_1(m+n-1)]\ (a_1+a_2)/(m+n)^2\ Ta \\
& + 2mb_1(n-1)(a_1+a_2)/(m+n)^2\ Tb \\
& - 2m^2b_1(a_1+a_2)/(m+n)^2\ Tc \\
& + (n-1)b_1/(m+n)^2\ TaTb \\
& - mb_1/(m+n)^2\ TaTc \\
& - m(n-1)b_1/(m+n)^2\ TbTc \\
& - (m+n-1)(m+n+1)b_1/2(m+n)^2\ Ta^2 \\
& + (n-1)^2b_1/2(m+n)^2\ Tb^2 \\
& + m^2b_1/2(m+n)^2\ Tc^2 \qquad \text{(B8)}
\end{aligned}$$

Note that the level of welfare is now expressed only in terms of the three tariffs. To simplify notation, it can be written as

$$\begin{aligned}
Ua = {} & a + bTa + cTb + dTc + eTaTb + fTaTc \\
& + gTbTc + hTa^2 + iTb^2 + jTc^2 \qquad \text{(B9)}
\end{aligned}$$

From the viewpoint of an individual exporter, the process is similar. Since all of the importers are identical, they would impose a tariff Tf and their demand can be expressed as

$$Qd = (n)a_1 - (n)b_1(P + Tf) \qquad \text{(B10)}$$

The supply by one individual country is written

$$Qs = a_2 + b_1(P - Td) \qquad \text{(B11)}$$

while the supply from the other (m-1) identical countries is written

$$Qs = (m-1)a_2 + (m-1)b_1(P - Te) \qquad (B12)$$

Combining the supply from equations (B11) and (B12), and equating it to the world demand in equation (B10) generates an equilibrium world price

$$Pw = (na_1 - m_2)/b_1(n+m) + [(m-1)Te + Td - nTf]/(n+m) \qquad (B13)$$

Substituting this value into equation (B11) yields the level of exports from this particular country

$$Qd = n(a_1+a_2)/(n+m) + b_1[-(n+m-1)Td+(m-1)Te-nTf]/(n+m) \qquad (B14)$$

Observe that the welfare approximation using producers' surplus and tariff revenue is given by

$$Ud = (1/2b_1)Qd^2 + TdQd \qquad (B15)$$

The important point to note is that the expression for welfare gained by an exporter is symmetric to the welfare expression for an importer. Hence, it is only necessary to consider one type of country, and the results will apply to the other type with some minor changes.

Since welfare for one importing country (country a) is a function of the three tariffs, it should be possible to maximize the welfare by treating Ta as a control variable. The primary problem arises when trying to model the responses of the other tariffs to changes in Ta. The dynamic programming method presented here starts with country A trying to maximize an objective function, given some belief about how the tariffs imposed in the next period (Tb and Tc) will change in response to tariffs in the current period. Time periods are denoted (t). The control variable is Ta, which is set by country A. The other two tariffs represent variables outside the direct control of country A and define the state of the system at any point in time. The system equations which define the movement of the two state variables are given as

$$Tb_{t+1} = \mu + \alpha Ta_t + \beta Tc_t \tag{B16}$$

$$Tc_{t+1} = \varepsilon + \gamma Ta_t + \theta Tc_t \tag{B17}$$

These two equations represent beliefs by country A about the manner in which the other countries will respond to changes in tariffs. Note first that the equations are linear. This linearity comes about because of the quadratic nature of the objective function, and will be shown to be sufficient below. Second, Tb_t does not appear explicitly in the equations. It does appear indirectly since Ta_t and Tc_t will be shown to depend directly on Tb_t. Hence, if Tb_t were to appear in equations (B16) and (B17), some of the equations generated below would not be independent.

The value of the system at any point in time is given by

$$K(Tb_t, Tc_t) = \max\{U(Ta_t, Tb_t, Tc_t) + \delta K(Tb_{t+1}, Tc_{t+1})\} \tag{B18}$$

Equation (B18) is basically the value of the current surplus, plus the value of any future surplus discounted by the factor (δ). Since the current welfare function is quadratic in three variables, it will be seen that the function K is quadratic in the two state variables, hence, let

$$K(Tb_t, Tc_t) = BTb_t + CTc_t + DTb_t^2 + ETb_tTc_t + FTc_t^2 \tag{B19}$$

The system is now fully expressed by equations (B16) through (B19). Assuming (initially) that the reaction coefficients in equations (B16) and (B17) are known, it is possible to find steady state values for the coefficients in equation (B19), hence the system will be solved. Once this function has been found, the optimal value of the control variable (Ta_t) is given as a function of the two state variables: Tb_t and Tc_t. Using equations (B16) and (B17), Ta_{t+1} can be expressed as a function of Tc_t and Ta_t which will be symmetric to equation (B17); and as a function of Tb_t and Tc_t which will be symmetric to equation (B16). This symmetry will then generate the equilibrium values of the reaction coefficients in equations (B16) and (B17).

Following the procedure outlined above, it is first necessary to find the optimal value of the control variable, which is found by differentiating equation (B18) with respect to Ta_t, and setting it equal to zero, generating

$$\begin{aligned} 0 = {} & b + eTb + fTc + 2hTa \\ & \delta[(B+2DTb_{t+1}+ETc_{t+1})\alpha \\ & + (C+ETb_{t+1}+2FTc_{t+1})\gamma] \end{aligned} \tag{B20}$$

where the (t) subscripts on the tariffs have been dropped for clarity. This equation reduces to

$$\begin{aligned} 0 = {} & b + \delta(\alpha B+pC) + \mu\delta(2\alpha D+\gamma E) + \varepsilon\delta(\alpha E+2\gamma F) \\ & + eTb \\ & + [2h + \alpha\delta(2\alpha D+\gamma E) + \gamma\delta(\alpha E+2\gamma F)]\ Ta \\ & + [f + \beta\delta(2\alpha D+\gamma E) + \theta\delta(\alpha E+2\gamma F)]\ Tc \end{aligned} \tag{B21}$$

This equation can now be solved for an optimal Ta

$$Ta_t^* = -(H/Y) - (R/Y)Tb_t - (S/Y)Tc_t \tag{B22}$$

where H, S, and Y are defined from equation (B21).

Using equations (B16) and (B17), (B22) can be written

$$Ta_{t+1} = -(H+\mu e+\varepsilon S)/Y - (\alpha e+\gamma S)/Y\ Ta_t$$
$$-(\beta e+\theta S)/Y\ Tc_t \tag{B23}$$

Note that this equation is symmetric to equation (B17). Combining this equation and equation (B22)

$$Ta_{t+1} = (\varepsilon-\theta H/Y) - (\theta e/Y)\ Tb_t$$
$$+ (-\theta S/Y + \gamma)\ Tc_t \tag{B24}$$

which is symmetric to equation (B16), since all importing countries are identical.

If the state equation (K) is known, the symmetry conditions yield solutions for the reaction coefficients. That is

$$\varepsilon = -(H + \mu e + \varepsilon S)/Y \tag{B25}$$

$$\gamma = -(\beta e + \theta S)/Y \tag{B26}$$

$$\theta = -(\alpha e + \gamma S)/Y \tag{B27}$$

$$\mu = \varepsilon + \alpha H/e \tag{B28}$$

$$\alpha = -\theta e/Y \tag{B29}$$

$$\beta = -\theta S/Y + \gamma \tag{B30}$$

The next step is to find the state function (K). That is, it is necessary to find equations that can be solved for the coefficients of equation (B19). These coefficients are found by assuming that the reaction coefficients are already known, and differentiating the objective equation with respect to the two state variables (Tb_t and Tc_t). Differentiating both sides of equation (B18) with respect to Tb_t

$$\partial K/\partial Tb_t = \partial Ua^*/\partial Tb_t + \delta \left[\frac{\partial K}{\partial Tb_{t+1}} \frac{\partial Tb_{t+1}}{\partial Tb_t} + \frac{\partial K}{\partial Tc_{t+1}} \frac{\partial Tc_{t+1}}{\partial Tb_t} \right] + \left[\frac{\partial K}{\partial Ta^*} \frac{\partial Ta^*}{\partial Tb_t} + \frac{\partial Ta^*}{\partial Tc_t} \frac{\partial Tc_t}{\partial Tb_t} \right] \quad \text{(B31)}$$

Using equation (B19), and equating the constant terms on both sides of the equation, equation (B31) implies

$$B = c + H \quad \text{(B32)}$$

Similarly, equation (B18) can be differentiated with respect to Tc_t. However, the dK/dTa^* term disappears by the envelope theorem. Hence,

$$\begin{aligned} C+ETb+2fTc = {} & d + \delta\theta C + \beta\delta B + \mu(\delta\theta E+2\beta\delta D) \\ & + \varepsilon(2\theta\delta F+\delta\beta E) \\ & + (S)Ta^* \\ & + (g)Tb \\ & + [2j + \beta(\theta\delta E+2\beta\delta D) \\ & + \theta(2\theta\delta F+\beta\delta E)]\ Tc \end{aligned} \quad \text{(B33)}$$

Note that this equation can be reduced to an expression of Ta^* as a function of Tb_t and Tc_t. Since there can be only one functional form of the equation, the constant term must be equal to the constant term in equation (B22), hence,

$$SH/Y = d + \delta\theta C + \beta\delta B + \mu(\delta\theta E + 2\beta\delta D) \qquad \text{(B34)}$$

Observe that this equation can be solved for the variable C as a function of B, D, and E.

To find a value for (D), equation (B31) can be differentiated with respect to Tb_t again, leaving

$$2D = 2i + e(-e/Y) \qquad \text{(B35)}$$

Similarly, differentiating equation (B33) with respect to Tc_t yields

$$2F = S(-S/Y) + (2j + \beta(\theta\delta E + 2\beta\delta D) + \theta(2\theta\delta F + \beta\delta E)) \qquad \text{(B36)}$$

Finally, using either equation (B31) or equation (B33) and differentiating by the appropriate tariff generates

$$E = g + S(-e/Y) \qquad \text{(B37)}$$

Hence, there is a system of equations which can be solved to yield the function K in terms of the reaction coefficients. Since the reaction coefficients are also expressed as functions of the coefficients of K, there is now a system of eleven non-linear equations and eleven variables. Fortunately, this system can be separated into two sets of equations: seven equations involving D, E, F, α, β, γ, and θ; and four equations involving the above values, plus B, C, μ, and ε. Unfortunately, only the second set can be solved analytically (once the other values are known).

However, the first set can be reduced to equations that yield interesting information. They can also be solved numerically to show some of the interrelationships between the optimal tariffs, and the number of exporters and importers in the system.

Consider equations (B26), (B27), (B29), and (B30). Equation (B29) generates a nice description of Y, and equation (B30) can be solved for S/Y. Substituting this latter value into equation (B26), and simplifying, yields an expression for (p) as a function of β, α, and θ. Similarly, equation (B27) can be written so that θ^2 is a function of α^2, β, and γ. Putting these two equations together

$$\gamma^2 = (\alpha + \theta)^2 \tag{B38}$$

Substituting this result back into the expression for β from equation (B30)

$$\beta = \pm 2\theta \tag{B39}$$

Before looking at the overall solution, some interesting observations can be made about these two equations. First, equation (B38) is very similar to the results presented in Chapter 3. That is, the exporter's reaction is equal to (or the opposite of) the reaction of the importer (or importers in this case). Second, the reaction by country B to country C is seemingly twice that of country A. However, it only appears that way, since Ta_t appears in equation (B16) and not Tb_t, so the factor of two takes into account the simultaneity mentioned above. Finally, note that the Cournot case is also a solution of the equations. In this case, α, β, γ, and θ are all equal to zero.

A great deal of analysis can be conducted with these four main equations, but the system cannot be solved analytically. However, the equations relating D, E, and F can be reduced to functions of the four reaction coefficients from equations (B35), (B37), and (B38). The basic idea is to start with guesses for α and θ, and use these guesses to generate new values for α and θ which are (hopefully) closer to the true values. There are many algorithms to solve systems of nonlinear equations, however, these functions appear to be extremely unstable, so it is necessary to use a process of bisection over two variables. Given the initial guesses of α and θ, (B38) gives a value for γ, (B39) gives a value for β. Y and S can be derived from equations (B29) and (B30). Values for D, E and F are found sequentially from equations (B36), (B37) and (B38). New values for Y and S are then calculated from their definition in equation (B21). Finally, zeros for each variable are created by subtracting the right hand sides in equations (B27) and (B29). Of course, equation (B38) generates two values (positive and negative) for γ.

Once the values of these variables are found, it is fairly straight-forward to solve the second set of equations. Equation (B25) can be solved for

$$\varepsilon = \mu(Y - e\theta)/(Y\theta + Y + S\theta) \qquad \text{(B40)}$$

Note that ε is related to μ by a "constant" multiplier, (call it V) in the sense that values for those variables have already been found. Hence, from equation (B28)

$$H = \mu(1 - V)Y/\theta \qquad \text{(B41)}$$

The value for B follows easily from equation (B32). C is derived from equation (B34). The equations can be simplified by making the following substitutions:

$$Za = \delta(2\alpha D + \gamma E)$$

$$Zb = \delta(\alpha E + 2\gamma F)$$

$$Zc = \delta(\theta E + 2\beta D)$$

$$Zd = \delta(2\theta F + \beta E) \qquad \text{(B42)}$$

Using these definitions, the system can be solved. For clarity, define

$$Zw = (1-\alpha\delta)(1-V)Y/\theta + (Za+VZb)$$

$$Zx = (\beta\delta Y-S)(1-V)/\theta - (Zc+VZd)$$

$$Zy = (d+\beta\delta c)/(\theta\delta-1) - (b+\alpha\delta c)/\gamma\delta \qquad \text{(B43)}$$

Hence,

$$\mu = Zy/[Zw/\gamma\delta + Zx/(\theta\delta-1)] \qquad \text{(B44)}$$

The values for ε, B, C, and H follow by substituting back into the previous equations.

Now that all of the coefficients have been found, it is possible to find the steady state values of the three tariffs. In steady state equilibrium, the tariffs remain constant over time, so the time subscript will be ignored. Substituting the resulting equation (B17) into equation (B16) expresses Tb as a function of Ta

$$Tb = \mu+\beta\varepsilon/(1-\theta) + [\alpha+\beta\gamma/(1-\theta)]\ Ta \qquad \text{(B45)}$$

Similarly, when substituting (B17) into the optimal tariff for country A found in equation (B22), let

$$Xa = 1 + (S/Y)\gamma/(1-\theta) \qquad \text{(B46)}$$

Then,

$$Ta = (-H/Y - (S/Y)\varepsilon/(1-\theta))/Xa - (e/Y)/Xa\ Tb \qquad \text{(B47)}$$

Combining equation (B46) and (B47) yields the steady state value for the optimal tariff

$$Ta = (-H/Y - (S/Y)\varepsilon/(1-\theta) - (e/Y)(\mu + \beta\varepsilon/(1-\theta)) \,/\, (Xa + (e/Y)(\alpha+\beta\gamma/(1-\theta)) \qquad \text{(B48)}$$

The other tariffs can then be found from equations (B45) and (B17).

APPENDIX C. ALGORITHM TO FIND MULTICOUNTRY TARIFFS

```
program dissmn(input,output);

uses transcend;

var alpha,beta,gamma,theta,mu,epsilon,
    s,y,hl,mlg,b0,cl,dl,el,fl,a,b,c,d,e,f,
    g,h,i,j : real;
    delta,al,a2,bl,pl,p2: real;
    ax,bx,cx,dx,erl,er2 : real;
    xll,xlh,x2l,x2h,xlinc,x2inc,cut : real;

    np,m,n,npl,np2 : integer;

    tariffa,tariffb,tariffc : real;

    setl,er,prbsl :boolean;

    yb : char;

procedure funl(xl,x2:real; var yl,y2:real);

var tl,t2: real;

begin

  er := true;

  alpha := xl;
  theta := x2;

  gamma := mlg * (theta + alpha);
  beta :=  2*mlg * theta;

  if (alpha = 0) or (theta = 0) then exit(funl);

  y := -theta*e/alpha;
  s := (gamma-beta)*y/theta;

  dl := i - e*e/(2*y);
  el := g - e*s/y;

  tl := -s*s/(2*y) + j + beta*theta*delta*el
        + beta*beta*delta*dl;
```

```
  t2 := 1-theta*theta*delta;

  if t2 = 0 then exit(fun1);

  f1 := t1/t2;

  t1 := (2*alpha*d1 + gamma*e1)*delta;
  t2 := (alpha*e1 + 2*gamma*f1)*delta;

  y := 2*h + alpha*t1 + gamma*t2;
  s := f + beta*t1 + theta*t2;

  if y = 0 then exit(fun1);

  y2 := theta + (alpha*e + gamma*s)/y;
  y1 := alpha + theta*e/y;

  er := false;

end; (* fun1 *)


procedure fun2;

var t1,t2,t3,t4,q1,q2,q3,q4,q5 :real;

begin

  q1 := (y-e*theta)/(y*theta + y + s*theta);

  q2 := delta*(2*alpha*d1 + gamma*e1);
  q3 := delta*(alpha*e1 + 2*gamma*f1);

  q4 := delta*(theta*e1 + 2*beta*d1);
  q5 := delta*(2*theta*f1 + beta*e1);

  t1 := (1-alpha*delta)*(1-q1)*y/theta + q2+q1*q3;
  t1 := t1/(gamma*delta);

  t2 := (beta*delta*y -s)*(1-q1)/theta - q4-q1*q5;
  t2 := t2/(delta*theta - 1);

  t3 := (b + alpha*delta*c)/(gamma*delta);

  t4 := (d + beta*delta*c)/(delta*theta-1) - t3;
```

```
  mu := t4/(t1 + t2);

  epsilon := mu*q1;

  c1 := -t1*mu -t3;

  h1 := -(mu - epsilon)*y/theta;

  b0 := c + h1;

end; (* fun2 *)


procedure eval(x1,x2:real; var y1,y2:real);

begin

 if set1 then fun1(x1,x2,y1,y2)

 else fun2;

end; (* eval *)


procedure bisect1(np:integer; er,vf,aq,bq:real;
                  var p1,tx:real);

var a1,b0,ta,tb,tp,e1,p :real;
    i : integer;

begin
  a1 := aq; b0 := bq; i := 0;
  eval(a1,vf,ta,tx);
  eval(b0,vf,tb,tx);

  repeat

    p1 := (a1+b0)/2;
    eval(p1,vf,tp,tx);
    e1 := abs(tp); i := i+1;

    if (prbs1) then  writeln(i,' ',p1,' ',tp);
    if tp*ta >0 then
    begin
      a1 := p1; ta := tp;
    end (* 1st *)
```

```
    else
    begin
      b0 := p1; tb := tp;
    end; (* if *)

  until (el<er) or (i>np);

end; (* bisect1 *)



procedure bisect2(np1,np2:integer; er1,er2,aq,bq,cq,dq:real;
                  var p1,p2:real);

var cr,dr,tc,td,tp2,e2,t1,pz :real;
    i : integer;

begin
  cr := cq; dr := dq; i := 0;

  bisect1(np1,er1,cr,aq,bq,p1,tc);
  bisect1(np1,er1,dr,aq,bq,p1,td);

  repeat

    p2 := (cr + dr)/2.0; pz := p2;

    bisect1(np1,er1,p2,aq,bq,p1,tp2);

    p2 := pz;

    e2 := abs(tp2);  i := i+1;
    writeln('* ',i,' ',p2,' ',tp2);

    if tp2*tc > 0 then
    begin
      cr := p2; tc := tp2;
    end (* 1st *)

    else
    begin
      dr := p2; td := tp2;
    end; (* if *)

  until (e2<er2) or (i>np2);

end; (* bisect2 *)
```

```
procedure initvars;

var t1,t2:real;

begin

  t1 := m/(m+n);  t2 := a1+a2;

  a := sqr(t1*t2)/(2*b1);
  b := (t1-2*m*b1*(m+n-1)/sqr(m+n))*t2;
  c := 2*m*b1*(n-1)/sqr(m+n)*t2;
  d := -t2*2*b1*sqr(t1);
  e := ((n-1)/(m+n) -(m+n-1)*(n-1)/sqr(m+n))*b1;
  f := ((m+n-1)*m/sqr(m+n) - t1)*b1;
  g := -b1*(n-1)*t1/(m+n);
  h := (sqr(m+n-1)/(2*sqr(m+n))-(m+n-1)/(m+n))*b1;
  i := (sqr(n-1)/(2*sqr(m+n)))*b1;
  j := sqr(t1)/2 *b1;
  mu := 0; epsilon := 0;
  h1 := 0;  b0 := 0; c1 := 0;

end; (* initvars *)

procedure results;

begin

  writeln; writeln;

  writeln('mu = ',mu); writeln('alpha = ',alpha);
  writeln('beta = ',beta); writeln('gamma = ',gamma);
  writeln('epsilon = ',epsilon); writeln('theta = ',theta);
  writeln;
  writeln('b = ',b0); writeln('c = ',c1);
  writeln('d = ',d1); writeln('e = ',e1);
  writeln('f = ',f1);
  writeln('y = ',y);
  writeln('s = ',s);
  writeln('h = ',h1);

end; (* results *)
```

```
procedure lims(var ax,bx,cx,dx,er1,er2:real;
               var np1,np2:integer);

var ya : char;
    t1,t2,f1,f2 : real;

begin
  er1 := 0.0001; er2 := 0.0001;
  np1 := 35;    np2 := 35;

  writeln;
  write('print data during bisect1?');  readln(ya);
  if ya = 'y' then prbs1 := true;

  write('change error for inside?'); readln(ya);
  if ya ='y' then
  begin
    write('err = ',er1); readln(er1);
    write('ctr = ',np1); readln(np1);
  end; (* if *)

  write('change error for outside?'); readln(ya);
  if ya = 'y' then
  begin
    write('err = ',er2); readln(er2);
    write('ctr = ',np2); readln(np2);
  end; (* if *)

  repeat
    repeat
      write('low inside: '); readln(ax);
      write('high inside: '); readln(bx);
      write('low outside: '); readln(cx);
      eval(ax,cx,f1,t2);
      eval(bx,cx,f2,t2);

      if f1*f2 > 0 then writeln('***error');
      writeln('f1 = ',f1,' f2 = ',f2);
      write('continue?'); readln(ya);

      if ya = 'i' then
      begin
        results;
        write ('continue?'); readln(ya);
      end; (* if *)

    until ya = 'y';
```

```
    write('high outside: '); readln(dx);
    bisectl(npl,erl,cx,ax,bx,tl,fl);
    bisectl(npl,erl,dx,ax,bx,tl,f2);

    if fl*f2 > 0 then writeln('***error');

    writeln('fl = ',fl,' f2 = ',f2);
    write('continue?'); readln(ya);

    if ya = 'i' then
    begin
      results;
      writeln('continue?'); readln(ya);
    end; (* if *)

  until ya = 'y';

end; (* lims *)


procedure getvars;

var yb : char;

begin

  write('default vars '); readln(yb);
  if yb = 'y' then
  begin
    delta := 0.5; m := 2; n := 2; bl := 2;
    al := 10; a2 := -5; mlg := 1;

  end (* lst *)

  else
  begin
    writeln;
    write('dscnt factor = '); readln(delta);
    write('# sellers (m) = '); readln(m);
    write('# buyers (n) = '); readln(n);
    write('abs slope (bl) = '); readln(bl);
    write('demand intcpt = '); readln(al);
    write('supply intcpt = '); readln(a2);
    write('gamma mult. +-1 = '); readln(mlg);
  end; (* if *)

end; (* getvars *)
```

```
procedure tariffs;

var t1,t2,t3,t4 : real;

begin

  t1 := epsilon - theta*epsilon + gamma*epsilon;
  t2 := 1 - 2*theta + theta*theta - gamma*gamma;

  tariffa := t1/t2;

  t3 := epsilon/(1-theta);
  t4 := gamma/(1-theta);

  tariffc := t3 + t4*tariffa;

  tariffb := mu + alpha*tariffa + beta*tariffc;

  writeln;
  writeln('tariff a = ',tariffa);
  writeln('tariff b = ',tariffb);
  writeln('tariff c = ',tariffc);
  writeln;

end; (* tariffs *)


procedure bsct;

begin

  getvars;
  initvars;

  set1 := true;
  prbs1 := false;
  lims(ax,bx,cx,dx,er1,er2,np1,np2);
  bisect2(np1,np2,er1,er2,ax,bx,cx,dx,alpha,theta);

  fun2;

  results;

  tariffs;

end; (* bsct *)
```

```
procedure range;

var ya : char;

begin
  write('set two?'); readln(ya);
  if ya = 'y' then set1 := false;
  write(' low x1 : '); readln(x1l);
  write('high x1 : '); readln(x1h);
  write('incr    : '); readln(x1inc);

  writeln;
  write(' low x2 : '); readln(x2l);
  write('high x2 : '); readln(x2h);
  write('incr    : '); readln(x2inc);

  writeln;
  write('zero cutoff : '); readln(cut);

end; (* range *)

procedure cmpr(y:real; var t: char);

begin
  t := '+';
  if abs(y) < cut then t:= ',';

  if y < 0 then
  begin
    t := '-';
    if abs(y) < cut then t := '.';
  end; (* if *)

end; (* cmpr *)


procedure map;

var i,j : integer;
    x1,x2,y1,y2 : real;
    t1,t2 : char;

begin
  set1 := true;
```

```
  getvars;
  initvars;
  range;

  np1 := trunc((x1h-x1l)/x1inc + 0.5);
  np2 := trunc((x2h-x2l)/x2inc + 0.5);

  if np2 > 40 then np2 := 40;

  writeln('inside (vert): ',x1l,' to ',x1h,'  by ',x1inc);
  writeln('outside (hor): ',x2l,' to ',x2h,'  by ',x2inc);
  writeln('cutoff: ',cut,'  neg=.    pos=,');
  writeln;

  for i := 1 to np1+1 do
  begin
    x1 := x1h - (i-1) * x1inc;

    for j := 1 to np2+1 do
    begin
      x2 := x2l + (j-1) * x2inc;

      eval(x1,x2,y1,y2);

      t1 := ' '; t2 := ' ';

      if not er then
      begin
        cmpr(y1,t1);
        cmpr(y2,t2);
      end; (* if *)

      write(' ',t1,t2);

    end; (* for j *)

    writeln;

  end; (* for i *)

end; (* map *)
```

```
begin    (* PROGRAM BEGINS HERE *)

  repeat

    write('m)ap  b)isect, q)uit');
    readln(yb);

    if yb = 'b' then bsct;
    if yb = 'm' then map;

  until yb = 'q';

end. (* temp  dissmn *)
```